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THE
ASTROPHYSICAL JOURNAL
AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

VOLUME XLII OCTOBER 1915 NUMBER 3

THE REFLECTING POWER OF METALS IN THE ULTRA-VIOLET REGION OF THE SPECTRUM

By E. O. HULBURT

INTRODUCTION

Knowledge of the reflecting power of metals in the ultra-violet region of the spectrum is far from complete. It was the purpose of the present investigation to extend as far as possible into the region of shorter wave-lengths the curves of reflecting power for metals, and thus to obtain data which might be of service in designing apparatus for use in ultra-violet work, and which might also possess theoretical value. In the present work the reflecting powers of metals, for an angle of incidence of 18°, have been measured throughout the region 3800 to 1800 A by a direct method.

HISTORICAL

In 1902 Hagen and Rubens[1] published the results of their measurements on six of the more common metals—silver, gold, platinum, copper, nickel, steel—and on four alloys, in which the curves of reflecting power were carried out as far as wave-length 2500 A. The method used was a direct one, and consisted in

[1] *Annalen der Physik*, **1**, 352, 1900; **8**, 1, 1902.

observing the incident and reflected intensities of a monochromatic beam of light by means of a sensitive thermo-couple. Their table has stood as the final word on the subject.

Beyond wave-length 2500 A there have been within the knowledge of the present writer no direct determinations at all of the reflecting power of any substance.

In 1903 Minor[1] made katoptric measurements on four metals—copper, steel, cobalt, and silver. He used a photographic method, and was able to get measurements for two points beyond 2500 A, namely, at 2313 and 2265 A. The reflecting powers were calculated by means of Drude's well-known formula connecting the reflecting power, the refractive index, and the absorption coefficient of the metal.

In 1910 Meier,[2] using a method similar to that employed by Minor, determined the reflecting powers from katoptric measurements down to wave-length 2500 A for gold, nickel, iron, platinum, bismuth, zinc, selenium, and for several alloys.

The results of these three investigations comprise our total knowledge on this subject, for the observations of other experimenters have contributed nothing new.

METHOD

The principle of the method employed in the present investigation was very simple. Light from a source rich in ultra-violet light was resolved into a spectrum, and a small, nearly monochromatic, beam was isolated by the slit B, Fig. 1, of the spectrograph, and passed into the metal chamber shown in Fig. 1. The intensity of the direct beam was measured by a photo-electric cell F connected with an electrometer. The intensity of the beam when reflected was measured by the same cell. This was effected by swinging both the cell and the mirror from their original positions M and F to the new positions M' and F' respectively. The length of the optical path from the slit to the cell was kept constant for the two positions. Consequently the ratio of the two measurements gave the reflecting power of the mirror.

[1] *Annalen der Physik*, 10, 581, 1903. [2] *Ibid.*, 31, 1017, 1910.

APPARATUS

The spectrograph.—This consisted of the usual Rowland mounting of a very fine concave speculum metal reflecting grating (*C*, Fig. 1) of 50 cm focal length. The grating had a ruled surface 6×9 cm, and was ruled in this laboratory especially for the purpose of this investigation. Slit *A*, Fig. 1, back of which the source of light was placed, was fixed on a movable brass arm *CD*. This arm was constrained to move by a pin through a slot at end *C*, and a pivoted slot underneath *A* sliding on a curved brass track *BA*, so

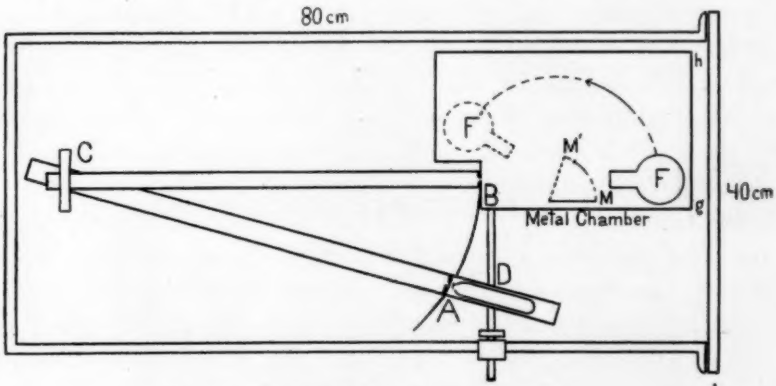


Fig. 1.—Horizontal plan of spectrograph

that slit *A* moved on the circumference of the circle whose diameter is *CB*. *CB* was 50 cm in length. Underneath the brass arm *AC* at *D* was a nut which worked without play on a steel screw (24 threads to the inch); by turning this screw the brass arm holding the slit *A* could be moved, thus allowing various wave-lengths to pass through slit *B*. Wave-lengths could be recorded by this apparatus throughout the region from 3800 to 1000 A.

The grating was ruled 15,000 lines to the inch by a diamond point so selected that the first-order spectrum on one side was very bright, and this bright first order was used throughout the present work. The spectrum, brought to a focus at slit *B*, was normal, and the dispersion of the grating was such that with slit *B* 0.5 mm wide a beam containing a wave-length range of 16 A passed through. In the present work slit *A* and slit *B* were both 0.5 mm wide.

The source.—An end-on hydrogen discharge tube, such as is described by Lyman,[1] was employed as the source of light. The tube was of the internal capillary type, equipped with a fluorite window and filled with hydrogen at about 1.5 mm of mercury pressure. This was excited by a 1100-volt transformer taking from 0.5 to 1.5 amperes in the primary, run on 60-cycle, 110-volt, alternating current. This tube, painted black, and wrapped with

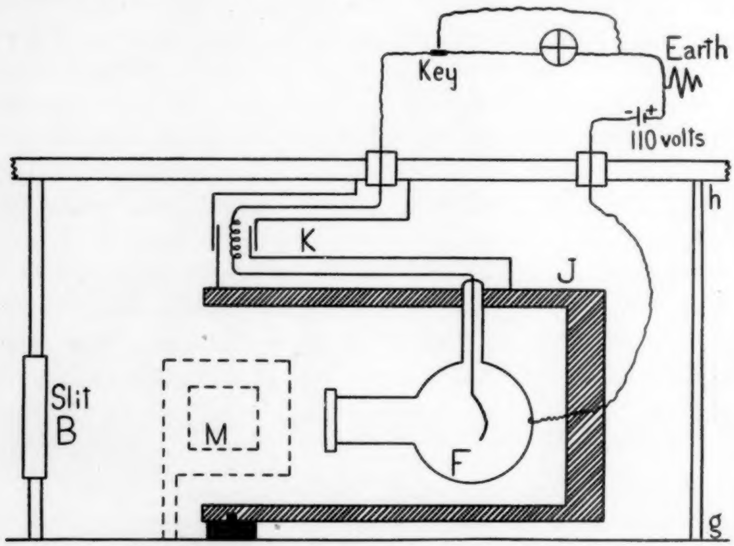


FIG. 2.—Metal chamber (elevation)

tin-foil radiators to keep it cool, was clamped in position back of slit A.

The photometer.—The light from the hydrogen tube was brought to a focus at slit *B*, and then passed into the metal chamber shown in horizontal section in Fig. 1 and in vertical section in Fig. 2. Inside this compartment was placed a photo-electric cell *F*, which, when connected with the electrometer, measured the intensities of the incident and reflected beams of light. The metal chamber, from which the end *hg* could be removed, served, when closed, to screen the photo-electric cell from all light, except the beam coming through slit *B*, and also to protect the cell and connections from

[1] *Astrophysical Journal*, **23**, 181, 1906.

electrostatic disturbances. The cell was supported in a brass frame J, rotating around a vertical axis. A second frame M (shown in dotted lines in Fig. 2) served as a support for the metal mirror. The mirror frame and the photo-electric cell frame were connected by a spring, so that when the cell was swung from F to F' the mirror swung from M to M' (Fig. 1). This whole shift could be made from the outside. The distance from the slit B to the mirror was 5 cm, and a mirror surface 4×2 mm was large enough to reflect a cone of light of sufficient size to completely cover the window of the cell at F'. The photometer consisted of the photo-electric cell and electrometer. The spectrum from the hydrogen tube was of very small intensity, even when the grating used was one of such short focal length and of such large area of ruling. Hence the instrument used to measure the intensity of the beam had to possess the utmost sensitiveness.

A cell in which sodium is the active metal is known to be very sensitive, and accordingly a method of preparing sodium photo-electric cells was developed,[1] and the cell finally selected was equipped with a fluorite window 14 mm in diameter and 1 mm in thickness. This sodium cell was wrapped in tin foil and placed in the supporting frame. The wire to the electrometer led from the cell through the jointed copper tube K, Fig. 2, out through sulphur supports to the earthing key, and thence to the electrometer. The electrical connections are shown in Fig. 2. The earthing key was merely a pointed brass rod touching a small brass plate, both made of the same piece of brass and filed bright. This gave no trouble with contact difference of potential. This type of key is, in the author's experience, much better than any mercury or electrolyte key, requiring absolutely no attention after it is once made, and giving complete satisfaction even for the most delicate work.

The electrometer was a Dolezalek quadrant electrometer with an aluminum needle. The needle was suspended by a quartz fiber rendered conducting by a slight coating of platinum put on by cathode sputtering, and was charged to 110 volts from a constant potential storage battery. In this work the electrometer had a sensibility of 3300 mm per volt difference of potential between the

[1] Hulburt, *Astrophysical Journal*, **41**, 400, 1915.

quadrants (the deflections being observed on a scale four meters distant), and in this case the needle had a free period of 21 seconds. The electrometer and the wire leading to the photo-electric cell were inclosed in metallic shielding made air-tight.

Through the kindness of Dr. E. Karrer, of the United Gas Improvement Company of Philadelphia, the writer secured the use of a potassium photo-electric cell which proved of the greatest service in the preliminary setting up of this apparatus.

In taking observations it was found most convenient to use the steady-deflection method, rather than a rate-of-drift method. The procedure was as follows: The connection to earth was broken, and the reading of the position of the electrometer needle recorded; then the hydrogen tube was turned on for a convenient length of time, e.g., 15 seconds, and the reading taken again after the needle had come to rest. If there was a natural drift of the needle—i.e., a drift when the cell was in the dark—the final reading was taken one minute after the first reading and the drift during the interval subtracted. During the course of the work there were times when the drift was zero and times when it amounted to as much as 10 mm per minute. The exact cause of the drift was not found, but the drift always increased with a rise in temperature of the room; probably this rise in temperature augmented the natural ionization of the air, and doubtless also produced effects in the photo-electric cell. The current through the hydrogen tube and the time of exposure were adjusted to produce deflections of convenient size; whenever possible, deflections for the direct beam of 100 to 200 mm were used.

The intensity of the light from the hydrogen tube fluctuated from day to day, but remained very constant for shorter periods of time. The changes in intensity were occasioned by changes in pressure inside the hydrogen tube, which were produced by the absorption and evolution of gases by the electrodes. Also the tube became gradually weaker and weaker in the region from 2500 A down. This was due to a film being formed on the inside of the fluorite window by the discharge; this film was invisible, but upon cleaning the window and refilling the tube with hydrogen the original power of the tube was regained. However, a single tube was useful for a month.

The sensibility of the photo-electric cell did not change perceptibly in several months, and since at all times the intensity of the light falling on the cell was very feeble, no evidences of photo-electric fatigue were ever observed.

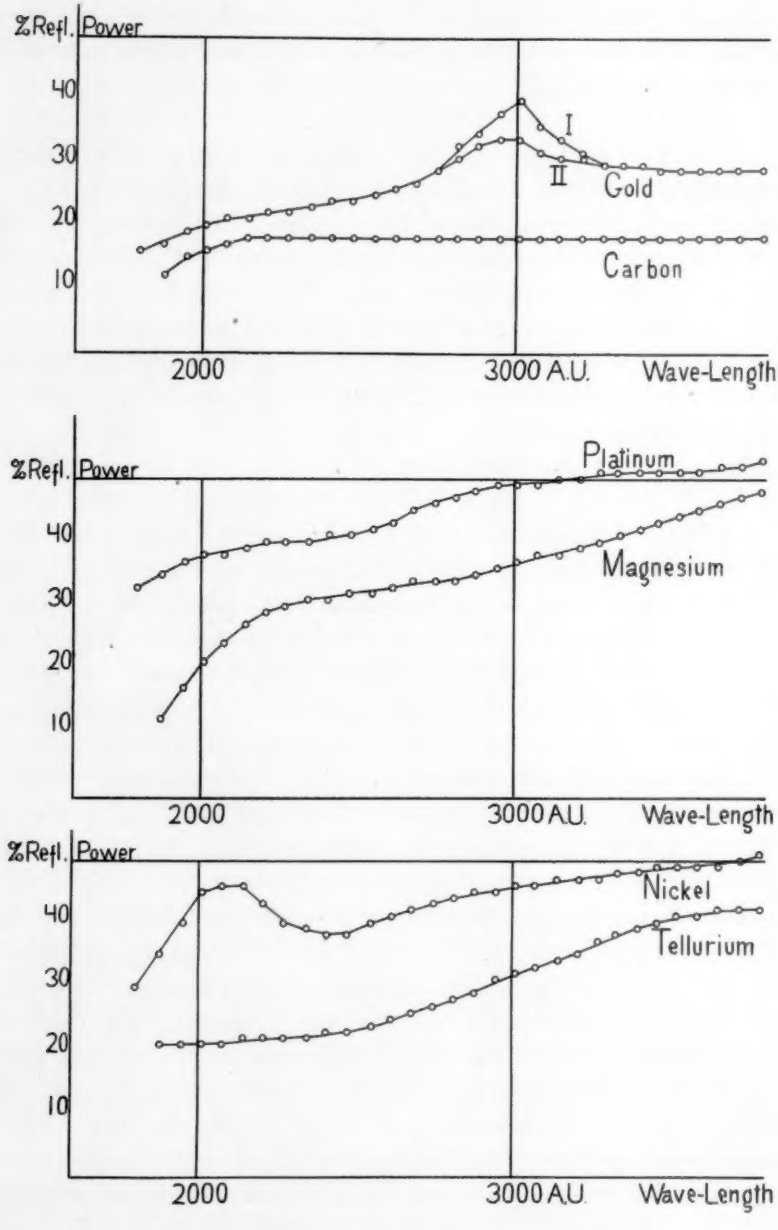
Measurements made with this apparatus on a polished quartz surface gave values of the reflecting power which agreed within the experimental error with the calculated values. Also the agreement with the results of other observers is satisfactory. Single observations varied by as much as 7 per cent, so the final values recorded in the work are the means of from two to six measurements. It is considered that the final values are correct to two parts in one hundred for reflecting powers above 25 per cent, the error increasing for lower values of the reflecting power.

RESULTS

The results are given in the form of curves, Figs. 3–7. The curves are plotted with the reflecting powers as ordinates and the wave-lengths in Ångström units as abscissae. The reflecting powers, for an 18° angle of incidence, were measured at intervals of about 60 A. In all cases where it is possible, comparison tables are given between the values for the reflecting powers determined by other workers and the values given in this paper; but it is to be noted that the reflecting powers found by others are for an angle of incidence 1–2°, or for normal incidence (when the reflecting power is calculated from katoptric measurements), whereas the values found in this work are all for an 18° angle of incidence.

Aluminum (Fig. 6).—Brilliant cathodic films of aluminum were produced by sputtering on glass from a freshly scraped aluminum cathode in an atmosphere of mercury vapor. It required about three hours to deposit an opaque layer of the metal. The curve is for an opaque film of aluminum. It is interesting to note that, with the exception of silicon surfaces, these films were the most efficient ultra-violet reflectors found.

Antimony (Fig. 4).—The mirror of Curve II was a cathodically deposited mirror from the Bureau of Standards, and although nearly free from imperfections, it did not seem as white as the aspect of the crystals of the metal would seem to indicate.



% Refl. Power
40
30
20
10
I
II
Gold
Carbon
2000
3000 A.U.
Wave-Length
% Refl. Power
Platinum
40
30
20
10
Magnesium
2000
3000 A.U.
Wave-Length
% Refl. Power
Nickel
40
30
20
10
Tellurium
2000
3000 A.U.
Wave-Length

FIG. 3

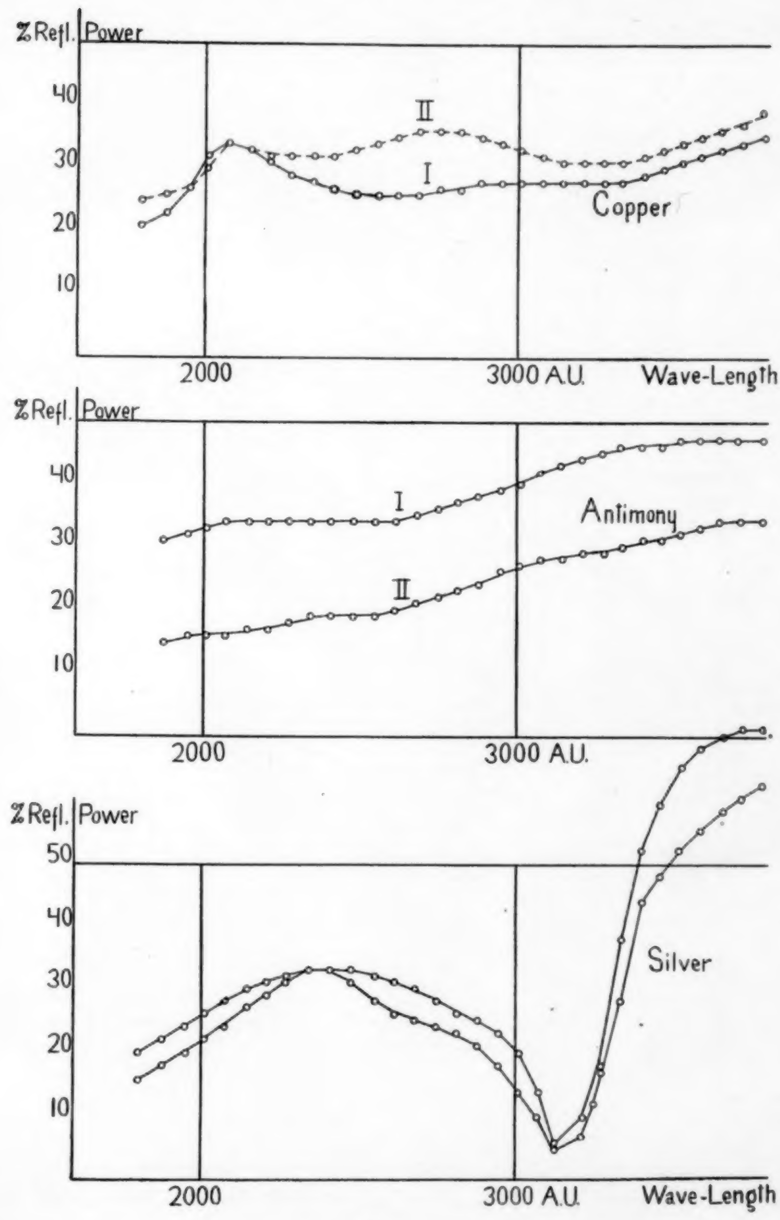

% Refl. Power
40
30
20
10
II
I
Copper
2000
3000 A.U.
Wave-Length
% Refl. Power
40
30
20
10
I
Antimony
II
2000
3000 A.U.
% Refl. Power
50
40
30
20
10
Silver
2000
3000 A.U.
Wave-Length

FIG. 4

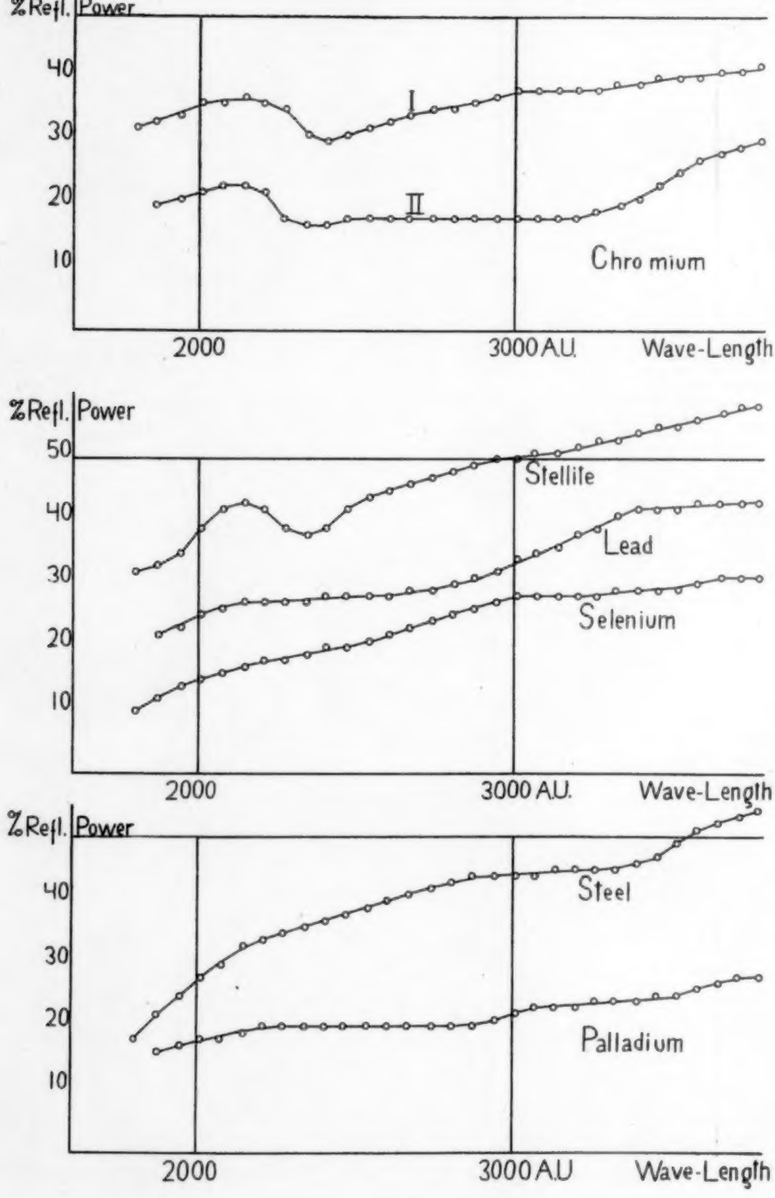
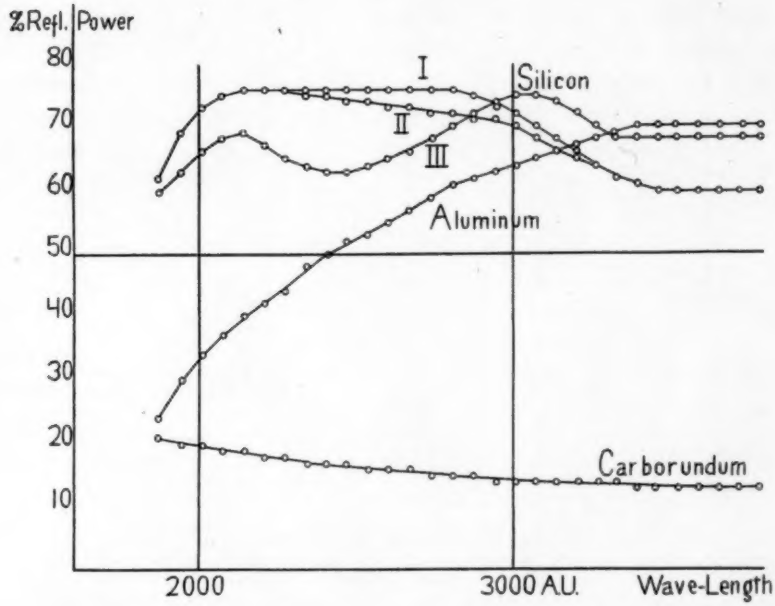

% Refl. Power
40
30
20
10
I
II
Chromium
2000
3000 A.U.
Wave-Length
% Refl. Power
50
40
30
20
10
Stellite
Lead
Selenium
2000
3000 A.U.
Wave-Length
% Refl. Power
40
30
20
10
Steel
Palladium
2000
3000 A.U.
Wave-Length

FIG. 5



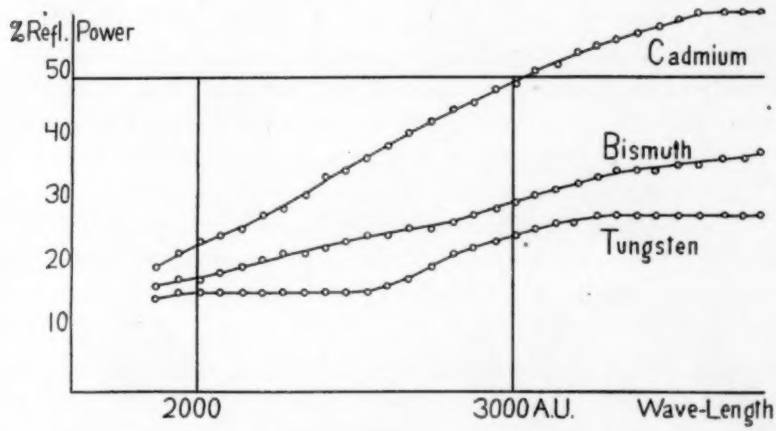
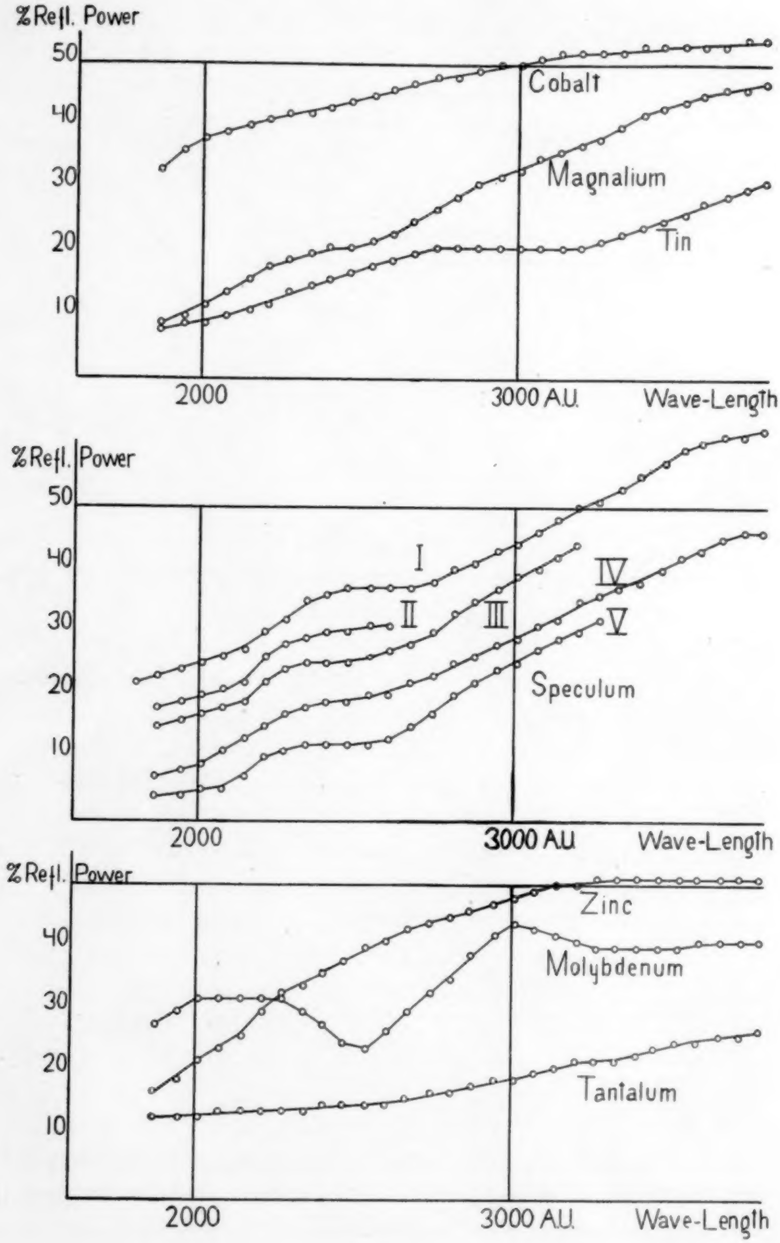


FIG. 6

A freshly split cleavage surface of a crystal, not perfect, however, but crossed by many cracks and crevices, resulted in Curve I. This did not deteriorate appreciably after standing for three weeks.



% Refl. Power
50
40
30
20
10
2000
3000 A.U.
Wave-Length
Cobalt
Magnalium
Tin
% Refl. Power
50
40
30
20
10
2000
3000 A.U.
Wave-Length
I
II
III
IV
V
Speculum
% Refl. Power
40
30
20
10
2000
3000 A.U.
Wave-Length
Zinc
Molybdenum
Tantalum

FIG. 7

Bismuth (Fig. 6).—This metal was found to sputter readily. The brightest surfaces were obtained when the deposition took place in an atmosphere of hydrogen. The curve is for an opaque film. Table I shows the comparison with results obtained by another observer.

TABLE I

REFLECTING POWER OF BISMUTH

Wave-Length	Meier Polished Plate	E. O. H. Cathodic
2573	20.1	25
2749	24.8	26
2981	31.2	29
3255	36.0	34
3611	42.5	36

Cadmium (Fig. 6).—This is a very white, soft metal and yielded bright films when sputtered cathodically in an atmosphere of hydrogen. Care had to be taken, as in the case of chromium, not to run the discharge too long at a time; for the yellow oxide very readily made its appearance, spoiling the deposit. The film was found to come down very quickly, a half-hour discharge being sufficient to produce opaque surfaces. These deposits were bright, but very soft; even brushing with a powder-puff produced scratches.

Carbon (Fig. 3).—Sputtering for four days from a piece of pure graphite produced a thin film of carbon slightly more opaque than the chromium film of Curve II (see chromium). In view of the fact that silicon possesses such remarkable reflecting properties in the ultra-violet, carbon, which is its near neighbor in the periodic system and which possesses similar electrical properties, might also be expected to show a high reflecting power in this same region. It is seen from the curve that the reflecting power of this carbon film is very low.

Carborundum (Fig. 6).—A perfect surface of a crystal was measured without further polishing. It was hoped that on account of its relation to silicon the carborundum might show peculiar reflecting properties. However, the curve for carborundum is of the same character as the curve for any dielectric, such as quartz. The

value of the reflecting power, 20 per cent at 2000 A, indicates a very high value of the refractive index.

Chromium (Fig. 5).—This metal is very white and is known to possess a high reflecting power in the visible region of the spectrum. It does not tarnish in air, and is not acted on by the ordinary reagents. Metallic chromium at present cannot be obtained free from small holes, and it was thought that cathodic films of this metal would be very serviceable, if they could be prepared.

The results are rather disappointing. Curve I is for an opaque film of chromium deposited cathodically on glass. The film was bright and free from imperfections, but showed a slight brownish tinge, which was undoubtedly due to traces of some impurity, probably the oxide or carbon. Curve II is for an extremely thin film, which, however, did not have the brown tinge, but was a very light gray by transmitted light. A solid plate of chromium was polished with pitch and rouge. The resulting surface was very bright but had many fine holes. The curve for this was practically the same as Curve I.

The curves show that surfaces of chromium are no better reflectors than films of many other metals which are much easier to prepare.

A well-defined minimum is noted at 2400, and a maximum at 2150 A. The alloy stellite also shows this same minimum and maximum, due no doubt to the presence of the chromium.

Chromium deposits extremely slowly and it was found difficult to obtain clean white deposits. The best films were obtained by sputtering in an atmosphere of hydrogen, running the discharge for a minute, and then allowing the cathode to cool for a few minutes. It required a week of such treatment to produce an opaque film.

Cobalt (Fig. 7).—A piece of rolled sheet cobalt was ground with emery, polished on pitch and rouge, and finally buffed. The surface took a good polish, being very bright, but showed many irregularities and corrugations. The reflecting power was surprisingly high, considering the appearance of the surface. Table II shows the comparison with results obtained by another observer.

Copper (Fig. 4).—The electrolytically plated gold mirror mentioned elsewhere was plated with copper from a solution of copper

cyanide. This was polished with rouge and chamois, giving a bright mirror, free from imperfections. Curve I is for this mirror.

TABLE II

REFLECTING POWER OF COBALT

Wave-Length	Minor Polished Plate	E. O. H. Polished Plate
2313	31.8	42
2573	39.7	45
2749	45.7	48
2981	48.7	50
3467	51.1	53

Measurements were made on two cathodic films of copper deposited on glass, one being more opaque and less red than the other. Both gave Curve II, which shows a more pronounced hump at 2800 A than other observers have recorded. Table III shows the comparison with results of other observers.

TABLE III

REFLECTING POWER OF COPPER

WAVE-LENGTH	HAGEN AND RUBENS POLISHED PLATE	MINOR ELECTROLYTIC	E. O. H.	
			Electrolytic	Cathodic
2313	....	29.0	29	32
2510	25.9	....	27	33
2573	....	27.9	26	34
2749	....	27.2	28	36
2880	24.3	....	29	36
2981	....	26.4	29	35
3050	25.3	....	29	33
3260	24.9	....	29	32
3467	....	31.5	31	33
3570	27.3	....	33	35

Gold (Fig. 3).—Several films of gold were deposited cathodically on glass. These deposits were very soft, and it was found that the character of the reflecting power changed after the mirror had been allowed to stand for some time in air. Curve I is the result of measurements on two different cathodic films of gold; these differed only in the thickness of the gold deposit, one appearing opaque and

the other quite green by transmitted daylight. They were measured a few hours after being made, and gave the same curve, Curve I, showing that the gold deposit in each case was thick enough to act as an entirely opaque mirror. The curve shows a sharp maximum at 3010 and a minimum at about 3700 A.

A second measurement of the two films two days later gave Curve II, and measurements a month later showed no further change in the curve. It is seen that the maximum in the curve for the older surface is no longer as sharply defined.

An electrolytic gold mirror was made by depositing gold from a solution of gold cyanide on a speculum mirror. This deposit was rubbed with rouge and chamois, and a bright surface free from scratches was obtained. The curve for this mirror, when new, was practically the same as the curves for the old cathodic films. Unfortunately this mirror was not kept; hence its behavior with time was not recorded.

The records for gold given by other observers do not agree with each other or with the values obtained in the present work. Table IV shows the comparison between the results of the various observers. Hagen and Rubens have not described how their mirror

TABLE IV

REFLECTING POWER OF GOLD

Wave-Length	Hagen and Rubens	Meier	E. O. H.	
			Curve I	Curve II
2570.......	38.8	27.6	25	25
2750.......	....	27.5	29	29
3050.......	31.8	30.4	40	33
3260.......	28.6	35.1	32	30
3570.......	27.9	37.7	30	29

was made. Meier measured a mirror of gold deposited electrolytically, and in his paper (*loc. cit.*) called attention to the fact that he did not find any indication of the minimum in the curve of reflecting power which Hagen and Rubens record at 3570 A.

Lead (Fig. 5).—This was found to sputter quite readily, the same precautions being necessary as were mentioned in the case of cad-

mium. The films are very soft, but come down bright; they soon tarnish. The curve is for an opaque film.

Magnalium (Fig. 7).—It was not found possible to produce a good mirror of this alloy. The specimen was a piece of the alloy made in this laboratory, 69 per cent aluminum and 31 per cent magnesium. The surface finally measured had many pits and scratches, and did not have a high polish. Table V shows the comparison with other results.

TABLE V

REFLECTING POWER OF MAGNALIUM

Wave-Length	Hagen and Rubens	E. O. H.
2510	67	21
2880	70	30
3050	72	34
3260	75	38
3570	81	45

Magnesium (Fig. 3).—A mirror was made of this metal by buffing and finally polishing with dry rouge and chamois. The resulting surface was bright, but had a number of fine scratches and corrugations.

Molybdenum (Fig. 7).—This was a polished specimen of the metal obtained from the Bureau of Standards. The surface was bright and free from scratches. The curve shows a small minimum at 2500 A.

Nickel (Fig. 3).—A fine mirror of nickel was obtained by first depositing a very thin cathodic film of nickel on glass, and then plating this electrically from a solution of nickel ammonium sulphate. This gave a very clean white opaque film, free from imperfections, and was measured a few hours after being made. The curve shows a well-marked maximum at 2110 A. Table VI shows the comparison with results obtained by other observers.

Palladium (Fig. 5).—This metal was deposited cathodically on glass, but the films thus made were not absolutely white; all showed a dark tinge. Several attempts to produce brighter deposits were unsuccessful, and it was decided that the color was due to the

nature of the metal rather than to an impurity or surface film. The curve is for an opaque film.

TABLE VI

REFLECTING POWER OF NICKEL

Wave-Length	Hagen and Rubens	Minor Electrolytic	E. O. H.
2510	37.8	30.9	38
2750		37.6	43
3050	44.2	39.4	46
3260	45.2	40.4	47
3570	48.8	41.2	49

Platinum (Fig. 3).—An opaque film of platinum was deposited cathodically on glass. Table VII shows the comparison with results of other observers.

TABLE VII

REFLECTING POWER OF PLATINUM

Wave-Length	Hagen and Rubens	Meier Electrolytic	E. O. H. Cathodic
2510	33.8		42
2573		37.1	43
2749		43.1	46
2880	38.8		48
2981		47.6	49
3050	39.8		49
3255		48.9	50
3260	41.4		50
3570	43.4		51
3611		52.4	52

Selenium (Fig. 5).—This was an old mirror of metallic selenium which had been prepared by melting the metal and pouring it on glass. For a complete description of the method see Pfund, *Astrophysical Journal*, **24**, 19, 1906. Table VIII shows the comparison with other results.

Silicon (Fig. 6).—A solid piece of silicon was ground with emery and finally polished with pitch and rouge. The resulting mirror was full of pits and had a number of coarse scratches. However, it possessed a remarkably high reflecting power.

A polished specimen of silicon from the Bureau of Standards was measured, and between 3000 and 2000 A it showed a reflecting power of 76 per cent (Curve I). The mirror was slightly convex and had a number of holes and scratches. A special sample of silicon free from holes was obtained from the Carborundum Company, Niagara Falls, and was polished by Dr. Anderson. This mirror gave Curve II. The reason for the difference between the two curves is not known. The two surfaces appeared very much alike; the one of Curve II seemed perhaps to have fewer imperfections.

TABLE VIII

REFLECTING POWER OF METALLIC SELENIUM

Wave-Length	Meier	E. O. H.
2573	23.3	21
2749	25.3	24
2981	31.8	28
3255	32.5	28
3570	30.3	30

Curve III is for an opaque cathodic film of silicon. The deposit was beautifully bright; its reflecting power for visible light more nearly approached that of fresh silver than any of the other metals investigated. In the ultra-violet it is seen that its reflecting power is somewhat inferior to that of the polished specimen of silicon.

The sputtering from the silicon cathode was performed in an atmosphere of mercury vapor, an aluminum anode being used, and it is quite possible that the silicon films contained aluminum. The curve (Curve III) for such a film also points to the possibility of the presence of aluminum, it being lower in the shorter wavelengths and higher in the longer wave-lengths than the curve for pure silicon. Subsequent experiment indicated that the percentage of aluminum present in these films was small.

Silver (Fig. 4).—A chemically deposited film of silver on glass, polished with rouge and chamois, was measured. The film was opaque, and was measured about a day after being prepared (Curve I). A three-quarter film of silver, showing a dark

gray-blue by transmitted light, gave Curve II. Table IX shows the comparison with other observers.

TABLE IX

REFLECTING POWER OF SILVER

WAVE-LENGTH	HAGEN AND RUBENS		MINOR	E. O. H. CURVE I
	Fresh	Old		
2263	....	....	18.4	32
2313	....	....	19.9	33
2500	....	....	25.0	33
2510	24.1	17.6	25.0	33
2880	21.2	14.5	19.0	25
3050	9.1	11.2	11.6	17
3160	4.2	5.1	4.2	6
3260	14.6	8.0	9.1	18
3380	55.5	41.1	61.7	49
3570	74.5	55.7	75.3	67

Speculum (Fig. 7).—Knowledge of the reflecting power of this alloy, 68.2 per cent copper and 31.8 per cent tin, is of particular interest because Rowland diffraction gratings are ruled almost universally on speculum. Curves I, II, and III illustrate the behavior of a speculum metal surface exposed to air. These curves were obtained from measurements on the same mirror. This mirror had a fine surface free from imperfections; it was freshly polished on pitch and rouge and was measured immediately, giving Curve I. Curve II was taken three days later; Curve III was taken seven days after the initial polishing. Table X shows the comparison with other observations.

TABLE X

REFLECTING POWER OF SPECULUM

Wave-Length	Hagen and Rubens	E. O. H. Curve I
2510	29.9	37
2880	37.7	41
3050	41.7	44
3570	51.0	60

In this laboratory, gratings are ruled entirely on speculum metal. It is the practice before ruling to polish the surface afresh on pitch

and rouge; hence a newly ruled grating finds itself in the condition of the mirror of Curve I. In subsequent use the grating either is never touched again, or at best is merely cleaned with chalk and alcohol. In order to find out definitely the efficiency of the usual grating, Curves IV and V were drawn; Curve V is for an old speculum mirror, with a surface highly polished but covered with a white film of tin oxide; Curve IV is for the same mirror after being cleaned with chalk and alcohol, all visible trace of the oxide being removed. Both curves show a lamentably low reflecting power, and this is undoubtedly due to the presence of the oxide film, which either is not entirely removed by the cleansing process, or immediately forms again.

These results show that there are possibilities for great improvement in the effectiveness of the grating for investigations in the ultra-violet. Depositing a thin layer of platinum or nickel on a speculum grating would increase its efficiency in the ultra-violet two or three times, and a layer of silicon would increase it as much as six times; it is planned to develop a method for doing this. Furthermore, the effect of a thin deposit on the lines of the ruled surface is to increase the brightness of the first order at the expense of the other orders. This is an additional advantage in all work in which the first order is the only one used, as in Lyman's investigation of the Schumann region and in the work recorded in this paper.

The behavior of a speculum surface in the Schumann region is unknown, but it is difficult to believe that the reflecting power experiences a marked increase in this region. That a grating yields such successful results as were obtained by Lyman in his photographic work in the region beyond 1800 A is surprising in view of the present record of the reflecting power of speculum.

Steel (Fig. 5).—The steel mirror was an old one of hardened steel, taken from a tuning fork. It had a fine polish with no imperfections, and was cleaned with chalk and alcohol. The comparison with other observations is shown in Table XI.

Stellite (Fig. 5).—The mirror was a polished specimen of the alloy obtained from the Stellite Works, Kokomo, Indiana. The mirror had a fine polish with no scratches. Stellite is an alloy of chromium and cobalt with impurities; the exact composition is a

commercial secret. As was mentioned under chromium, the curve for stellite shows a maximum at 2150 and a minimum at 2400 A, as do the chromium curves.

TABLE XI

REFLECTING POWER OF STEEL

Wave-Length	Hagen and Rubens Unhardened	Minor	E. O. H. Hardened
2265	....	34.8	35
2313	....	35.7	36
2510	32.9	38.9	38
2880	35.0	42.1	44
3050	37.2	42.9	44
3260	40.3	44.8	45
3570	45.0	50.8	50

Tantalum (Fig. 7).—This was a polished specimen of the metal, but the surface finally measured, while having a high polish, was marred by fine scratches. The sample was obtained from the Bureau of Standards.

Tellurium (Fig. 3).—This was a cathodic mirror from the Bureau of Standards. The film was opaque, bright, and free from scratches; it was rubbed with rouge and chamois before measuring.

Tin (Fig. 7).—An opaque surface of tin was deposited cathodically on glass. The mirror was bright and free from imperfections, but was measured three days after being made. It is likely that the surface became very quickly covered with a layer of oxide, which reduced the reflecting power materially. The oxide made its appearance unmistakably in a few days, appearing as a white film.

Tungsten (Fig. 6).—The piece examined was a polished specimen of the metal obtained from the Bureau of Standards. The surface had a good polish, but was marred by a few holes and fine scratches; it was rubbed with rouge and chamois before measuring.

Zinc (Fig. 7).—Bright films of zinc were obtained by sputtering in an atmosphere of hydrogen. The curve is for a three-quarter

opaque film. Table XII shows the comparison with results obtained by another observer.

TABLE XII

REFLECTING POWER OF ZINC

Wave-Length	Meier Polished Plate	E. O. H. Cathodic
2573	20.5	40
2749	47.6	44
2981	60.2	48
3255	68.2	51
3611	70.5	52

DISCUSSION OF RESULTS

The curves show two general characteristics: (1) in the region of the spectrum covered in this investigation the reflecting power decreases as the wave-length decreases; and (2) the reflecting power is never zero. A number of the curves have another characteristic in common. Beyond wave-length 2000 A many of them take a well-defined steeper slant; that is, the reflecting power begins to decrease much more rapidly. That this effect is due to some peculiarity of the apparatus is improbable, for the deflections in this region are very large, and the determination of the reflecting power is accurate even for quite low values of the reflecting power. Furthermore, a number of the substances do not show this rapid falling off in reflecting power. It may be that this observed decrease in reflecting power is a genuine property of the metal, or it may be that the air in contact with the reflecting surface plays an important part. Air has a strong absorption band which begins at 1630 A, and marked absorption starts to set in at 1900 A. It is possible that the optical constants of air begin to change rapidly from wave-length 2000 A down as the absorption band is neared, and this rapid change in the optical properties of the layer of air on the surface of the metal may cause a marked change in the reflecting power of the metal.

A certain similarity exists in the curves for copper, chromium, nickel, cathodic silicon, and molybdenum, for they all show a shallow minimum, i.e., a region of lower reflecting power, in approxi-

mately the same region of the spectrum. There is no evident cause to which this may be attributed. That it is due to the presence of some common impurity is improbable, because the methods of preparation of these five mirrors differed widely.

The one general conclusion that follows from the results of this investigation is that for the metals the reflecting power decreases as the wave-length of the light decreases. Whether this same relation continues to hold for wave-lengths beyond 1800 A in the Schumann and Lyman regions, can be determined only by future experiment. Metals, then, have a maximum reflecting power in the visible and infra-red regions of the spectrum, and it is concluded that the reflecting power diminishes on the short wave-length side of this maximum with no indication of ever increasing again.

Silicon is the one exception to the foregoing conclusion; the reflecting power rises to 76 per cent at 3000 A, remains practically constant from 3000 to 2000 A, and then beyond 2000 drops rapidly, reaching a value of 62 per cent at 1870 A. Future work may show that this rapid decrease beyond 2000 A continues until a low reflecting power is reached, and, if this be so, the elevation in the curve is a region of selective reflection.

Silicon differs from platinum, copper, etc., in that it is a poor conductor of electricity; it is a so-called "metalloid." The behavior of silicon suggests that perhaps there is a class of substances which have the maximum reflecting power in the extreme ultraviolet, just as the true metals show a maximum in the region of the spectrum of longer wave-length. None of the other substances of the metalloid class which were investigated, such as carbon, antimony, etc., show characteristics similar to silicon, and perhaps silicon is unique in this respect.

From a practical standpoint the curves show that from all the metals investigated (with the exception of silicon) two may be selected as being the most serviceable for use in work requiring the reflection of light of short wave-lengths: platinum and nickel. Optical surfaces of these two metals may be readily prepared by cathodic sputtering and by electroplating; these surfaces are quite hard, are slow to tarnish in air, and are not acted on by most of the common laboratory reagents.

Those metals which oxidize easily should be avoided. The appearance of a white or cloudy film on the surface is almost a sure sign of a very low reflecting power in the ultra-violet; on the other hand, the good ultra-violet reflectors give very clear white or bluish-white reflections in visible light.

In this connection silicon again deserves special mention. This metal possesses in a marked degree all the physical properties essential for a perfect reflecting surface; it is as hard as ordinary glass without being brittle; it is not attacked by weak or strong acids, and of course does not tarnish in air; at ordinary temperatures the only reagent which acts on silicon is concentrated potassium (or sodium) hydroxide. The metal takes a high polish by the ordinary method of polishing with pitch and rouge, and brilliant deposits of silicon can be obtained by cathode sputtering, as has been brought out in this investigation. The reflecting power of a silicon mirror was found to be 76 per cent in the region from 2000 to 3000 A (see silicon, Curve I, Fig. 6). The same mirror has been measured by Coblentz[1] in the region of longer wave-lengths, and its reflecting power in the green was found to be 34 per cent, dropping to 28 per cent in the infra-red. Hence a silicon mirror, or grating, possesses a high efficiency throughout the entire range of the spectrum, and the advantages of its use in optical instruments are obvious. Thin films of silicon on interferometer plates will undoubtedly enable investigations in this field to be extended down to 2000 A.

The preparation of mirrors and gratings upon silicon plates depends only upon the possibility of procuring large plates of the metal which are homogeneous; for all the large specimens examined have been found to be porous, and hence are not suitable for the best results. The task of casting silicon pure and in a homogeneous state is certainly not an insurmountable one, and is a problem for the commercial laboratory rather than for the scientific investigator.

SUMMARY

An ultra-violet spectrograph has been set up in which a sodium photo-electric cell connected to an electrometer serves to measure

[1] *Bull. Bureau of Standards*, 7, 217, 1911.

the intensity of the light. With this the reflecting powers of twenty-eight metallic mirrors have been examined; namely, Al, Sb, Bi, Cd, C, carborundum, Cr, Co, Cu, Au, Pb, magnalium, Mg, Mo, Ni, Pd, Pt, Se, Si, Ag, speculum, steel, stellite, Ta, Te, Sn, W, Zn.

The curves of reflecting power have been drawn throughout the region 1800 to 3800 A, and it has been shown that the reflecting power for light of wave-lengths shorter than 3000 A is rarely above 50 per cent, with one noteworthy exception, silicon, which shows a reflecting power of 76 per cent in the region 2000 to 3000 A.

It has been shown that brilliant opaque films of silicon can be prepared by cathode sputtering.

In conclusion I wish to express my hearty thanks to Professor Ames for his interest throughout the course of the work.

Dr. Pfund has been especially helpful; it was at his suggestion that the work was undertaken, and his advice and assistance at every stage have been invaluable. I am deeply indebted to Dr. Anderson for his kindly interest and many helpful suggestions.

Dr. W. W. Coblentz, of the National Bureau of Standards, has kindly allowed the use of several mirrors of the more uncommon metals.

Johns Hopkins University
June 1915

A STUDY OF THE POLE EFFECT IN THE IRON ARC[1]

By CHARLES E. ST. JOHN and HAROLD D. BABCOCK

I. INTRODUCTION

In the investigations made at this observatory upon the determinations of standards of wave-length and upon the relation between laboratory and solar spectra, it was early found that a detailed study of the iron spectrum would be required as a preliminary to further definite progress.[2]

The measurements by different observers of the wave-lengths of the iron lines that serve as standards in the international system show discrepancies which far exceed the limit of precision attainable with a grating spectrograph of high dispersion, and the interpretations of sun-arc displacements may be illusory and lead to diverse conclusions, if the presence and influence of lines of certain types are unrecognized. The divergent results were found to depend upon conditions in the arc, its length, the current-density, and the portion of the arc from which the light is taken, and to inhere in those types of lines that show more or less dissymmetry under varying pressure and line intensity and that appeared to have large or abnormal pressure displacements.[3]

The purposes in view in undertaking the investigation of these sensitive and unstable lines were: (1) to determine whether the variations in wave-length observed with different arc conditions are real or fictitious, that is, whether there are actual displacements of the maxima of the lines; (2) to determine whether there are general pressure differences in the arc sufficient to account for the displacements; (3) to examine some other conditions in the arc

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 106.

[2] *Mt. Wilson Contr.*, No. 75; *Astrophysical Journal*, **39**, 9–14, 1914.

[3] *Mt. Wilson Contr.*, No. 61, pp. 2–5; *Astrophysical Journal*, **36**, 15–18, 1912; *Mt. Wilson Contr.*, No. 75, pp. 5–8; *Astrophysical Journal*, **39**, 9–12, 1914; *Mt. Wilson Contr.*, No. 93, pp. 33–36; *Astrophysical Journal*, **41**, 61–64, 1915; Goos, *Astrophysical Journal*, **38**, 141, 1913.

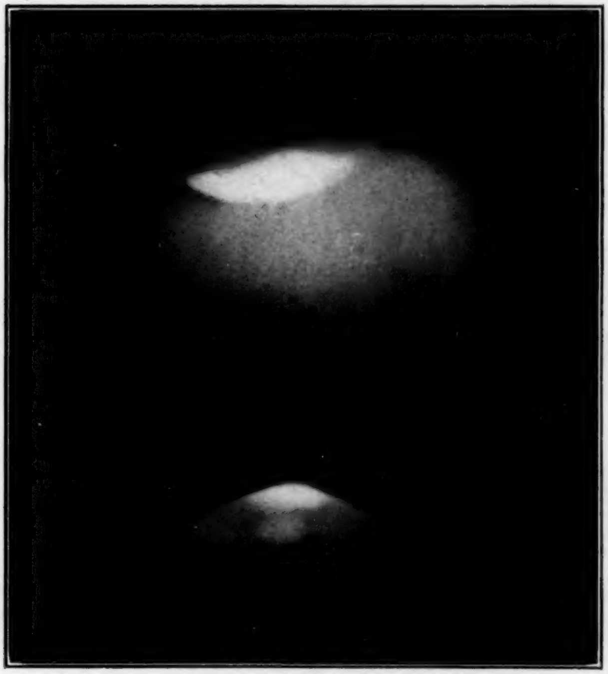
which may be effective; (4) to investigate a wide range of the spectrum for the identification and distribution of the questionable lines, which must necessarily play an important rôle in several fields of investigation; (5) to indicate working conditions which should be observed when lines of this character are to be used as standards of reference.

II. METHODS AND APPARATUS

Our method has involved an accurate comparison of the spectrum from near the poles with that from the center of the arc. In the effort to carry such a comparison of wave-length to the third decimal place, difficulty was encountered in obtaining plates free from line displacements of small but determinable amounts due to instrumental causes or observing conditions. When the problem is such that crucial tests can be applied to the results, the difficulties involved in obtaining comparison spectra of consistent reliability are quite impressive. These were in the end eliminated by making the exposures rigorously simultaneous.

A greatly enlarged image of the arc was thrown upon the slit of a plane-grating Littrow spectrograph of 30 feet focus by means of an achromatic lens and a large totally reflecting prism above the slit. Plate IV*a* shows an image of the arc as it actually appeared upon the slit. The light from the central portion passed directly to the slit, at *A*, Plate IV*b*, while a system of two small totally reflecting prisms, *B* and *C*, permitted light to be taken from any desired point on the axis of the arc. The spectrum of the light from this selected portion of the arc fell between the two narrow spectra of the equatorial section. For measurements of high precision, it is of great advantage to have the lines of the two spectra of nearly the same intensity, and this is particularly important when the lines have a tendency to broaden unsymmetrically. The relative exposure times were controlled by a rotating sector of variable opening placed above the first prism in the path of the light coming from near the pole of the arc, as shown in Plate IV*b*. The exposures began and ended at the same instant, one being continuous and the other rapidly intermittent. In this manner any effect due to slow changes of temperature, to flexures of apparatus, or to vibrations reaching

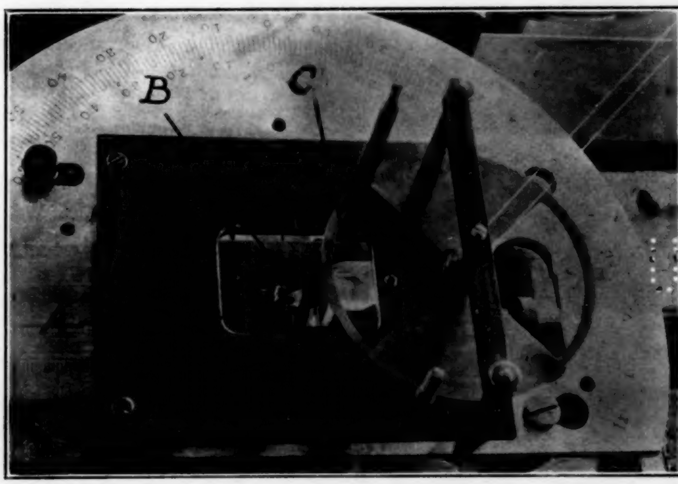
PLATE IV



−Pole

+Pole

a. THE ARC AS PROJECTED UPON THE SLIT



b. SECTOR AND PRISMS IN POSITION

the instrument through the ground was effectively eliminated; no part of the instrument was touched during the exposures, so that no effects due to mechanical displacements were introduced. Great care was used to secure complete illumination of the grating from each light-source. A similar arrangement was used in comparing arcs with different current-densities and it is applicable to all comparisons where the highest precision is sought. It will be added to the reconstructed 60-foot tower telescope equipment for sun-arc comparisons.

Before the sector was employed the exposures were made of unequal length in order to equalize the intensities. They were simultaneous during the period covered by the shorter; but, though the apparatus is very stable, was not touched during the exposures, and has its optical parts in an underground chamber at practically constant temperature, there were minute displacements, positive on some plates and negative on others, differing even for successive exposures on the same plate. It is not a sufficient precaution to divide an exposure of a comparison spectrum with the thought of detecting an instrumental displacement when the quantities involved are of the order of a thousandth of an angstrom.

The Pfund[1] form of arc, 6 mm long, carrying a current of 6 amperes, has been found to be a very stable and reliable source. In obtaining standards of reference the slit of the spectrograph was placed normal to the axis and in the central plane of the enlarged image of the arc. Except where otherwise noted the Pfund arc has been thus employed for all wave-length comparisons given in this paper.

Some plates in the ultra-violet were taken with a 5-inch concave grating of 15 feet radius. On account of astigmatism the arrangement of prisms and sector described above could not be employed. The exposures were therefore not simultaneous, and the use of a shutter in front of the plate became necessary. Under these conditions minute displacements due to instrumental causes appeared upon the plates, for the correction of which a series of overlapping exposures was made to connect these photographs with those obtained by means of the plane grating.

[1] *Astrophysical Journal*, **27**, 296, 1908.

In a valuable paper Eder and Valenta[1] call attention to the precautions necessary to obtain reliable photographs of spectra. The care that we have given to this question may be seen from the following details. The 4-inch plane grating (ruled surface 63×72 mm, 42,386 lines) used in the Littrow spectrograph of 30 feet focus was ruled by Anderson on the reconstructed Rowland machine. It has been shown to yield over 90 per cent of its theoretical resolving power in the second order, and is particularly well suited to the purposes of this investigation, as it produces bright line spectra of a high degree of perfection. In the second order, which was used for this work, the diffraction pattern is distinct and symmetrical on the two sides of a good line, and when the spectrum is observed visually the diffraction fringes of the lines of pressure groups *a* and *b* remain sharp and distinct even near the poles of the arc. The focus for any particular region is read from an experimentally derived curve which is accurate to 0.5 mm; the focal length of the instrument is 9.16 m. The slit-widths were always very nearly four times the Rayleigh normal, and give 80 per cent of the resolving power available with an indefinitely narrow slit and 77 per cent of the intensity given by a very wide one, thus making an excellent compromise between opposing conditions. Tests were made to detect a possible variation of the gradation-curves of the lines with the slit-width, but no effect was found for the widths employed. Great care was taken to avoid overexposing the spectrum lines. For the microphotometric measurements with the Hartmann and Koch instruments no lines were used whose maximum blackness was such as to lie in the region of overexposure, as defined by the characteristic curve for the plate. On the plates used for filar micrometer measurements the exposures were timed to equalize the widths and intensities at pole and center. If the strongest lines were possibly overexposed, the weakest lines on the same plate were underexposed, but among the lines of intermediate strength some must have had correct exposures. Lines of all ranges of intensity, however, showed the effect.

For obtaining an independent check upon the magnitude of the pressure-effect some photographs were made by means of an

[1] *Astrophysical Journal*, **19**, 251, 1904.

interferometer of the Fabry and Perot type, recently constructed in our instrument shop. The plates, by Hilger, are of fused quartz, 40 mm in diameter and 5 mm thick. Their surfaces are of the highest quality and in performance they equal our glass plates furnished by Jobin. We have coated them with silver by the method of cathodic sputtering *in vacuo*, obtaining perfect uniformity in the films and a reflecting power so high that more than 60 images of the sun can be counted through them. The étalon used is made of invar and has a thickness of 10 mm. The rings are projected by means of an achromatic objective of 41 cm equivalent focal length upon the slit of a 13-foot Littrow spectrograph. The auxiliary dispersion is produced by a 4-inch plane grating having a remarkably bright first-order spectrum.

III. THE REALITY OF THE DISPLACEMENTS

As the lines exhibiting these variations in wave-length show more or less dissymmetry under pressure and are greatly widened at the pole, it has been thought by some that the displacements in question are only apparent and arise from the difficulties inherent in the measurement of lines of this character, and by others that they are due to increase of pressure or of vapor-density in the arc. The point is not only of theoretical interest because theories of the production of spectrum lines must be founded upon detailed study of the conditions of emission, but it is also of importance in view of the possible employment of these lines in solar and stellar investigations, for which their peculiar characteristics promise to be of value, provided their properties can be well established and the conditions determined under which they may be used.

To eliminate as far as possible the disturbing effects due to dissymmetry, we have made the net exposure times such as to give practically equal intensities to the lines from the two parts of the source. The lines $\lambda\lambda$ 5424 and 6400 observed in this way are shown in Plate V*a*, with a twenty-eight-fold enlargement. These are typical lines, showing displacements toward the violet and red, respectively. In order to make the shifts still more apparent in the reproduction, fine artificial lines have been ruled through the centers of the comparison spectra. For an extreme case, the

relative exposures were such that the intensity and width of the lines in the spectrum from near the negative pole were less than for the same lines in the spectrum of the central section of the arc. In this case, lines that widen unsymmetrically to the red would not show at the pole a displacement toward greater wave-lengths having its origin in this quality of dissymmetry. An illustration is shown in Fig. 1, which is a diagram to scale from measurements of λ 6400 taken under these conditions. It is evident that if the width of the line, as observed near the pole, were increased by a longer exposure to equal that of the same line in the spectrum of the central section of the arc, the line would still show a distinct displacement to the red. When such lines are of moderate intensity upon the photographic plate, an experienced observer can make micrometer settings upon their maxima with surprising consistency.

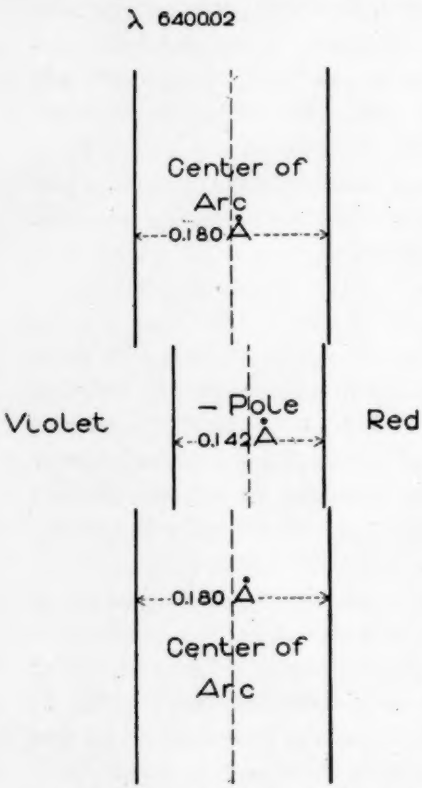
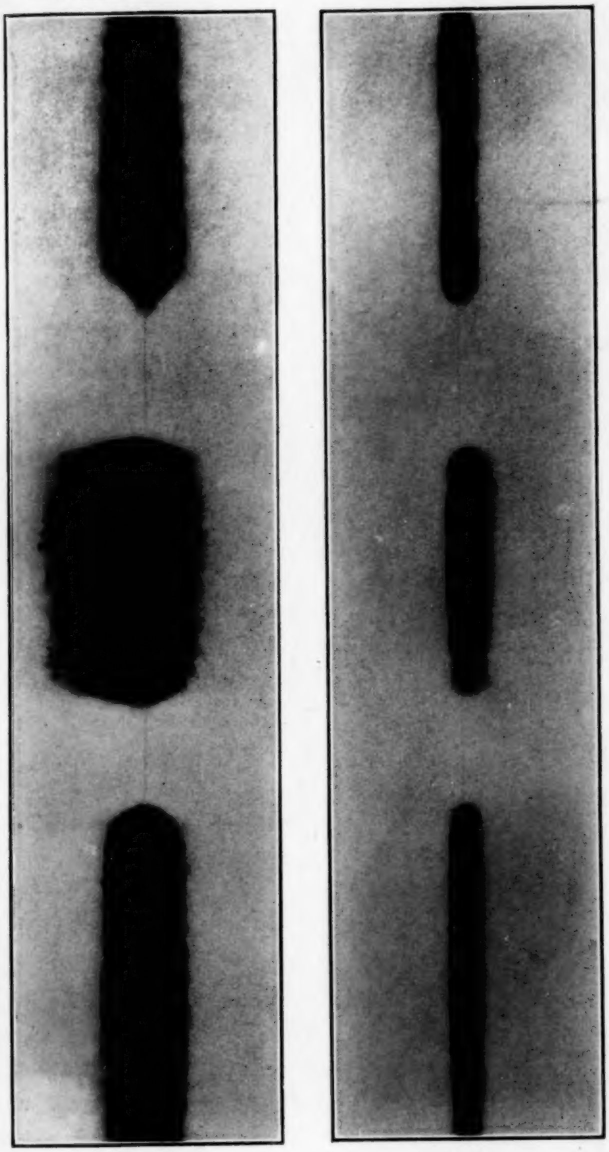


FIG. 1.—Persistence of displacement when width at pole is less than at center.

Dissymmetry is not evident in the case of every line that gives a displacement to the red at the pole of the arc as compared with its position at the center. For example, λ 5339 is apparently as good a line as λ 5341, though the first belongs to group *d* and the second to group *a*. On one plate, for illustration, the widths of the line at the center and negative pole are 0.106 and 0.108 A, respectively, and the measured shift of the latter is 0.015 A to the red, which cannot be accounted for on the ground of unsymmetrical widening when the increase in width is only 0.002 A.

PLATE V

Violet Red

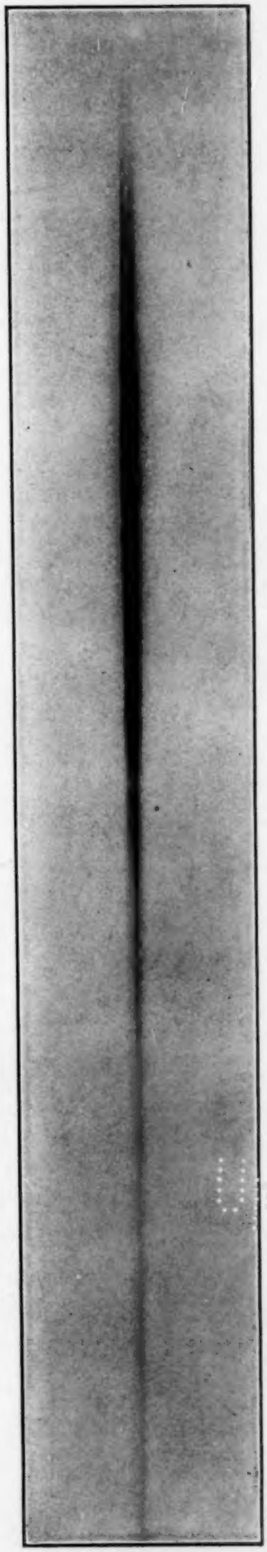


λ 5424 λ 6400

a

V R

— Pole



Center

b

a. Displacements at pole toward violet for λ 5424, toward red for λ 6400. Artificial lines ruled through centers of comparison spectra. Enlarged 28 times.

b. Iodine absorption line superposed upon iron line λ 5424, showing displacement of the latter at the negative pole. Enlarged 6.7 times.

In such cases the personal equation of the observer is involved, and one may question more or less the possibility of measuring lines of this character to the degree of precision suggested. In order to eliminate the personal element, measurements have been carried out with the Hartmann and Koch microphotometers. Fiducial lines were drawn upon the plate, generally one on each side and as nearly parallel as possible to the spectrum line under investigation. With the Hartmann microphotometer settings were made by 0.02 mm steps from one fiducial line to the other, across them and the spectrum lines along each of the three divisions of the spectrum. The density-curves were plotted to scale and the positions of the maxima determined relatively to the fiducial lines. Some lines of groups *a*, *d*, and *e* were measured in this manner, and in all cases the measurements showed displacements of the maxima of the *d* and *e* lines to the red and violet, respectively. Curves determined by the Hartmann microphotometer are shown in Fig. 2 for lines of groups *b* and *d*, and in Fig. 3 for lines of groups *a* and *e*. The results have been confirmed by the photographic records made with the Koch form of microphotometer. Both instruments show that the displacements of the maxima are unquestionable and yield the same values as those found by the usual method, but the dissymmetry is much less striking in the graphs than was expected from the visual examination of the lines. It would seem that the eye is very sensitive to contrast and overestimates the degree of asymmetry.

An entirely independent test for the reality of the displacements was made by superposing upon the spectrum of the iron arc the absorption spectrum of iodine vapor under low pressure. For this purpose the slit was placed parallel to the axis of the arc, the prisms and sector being removed, and a spherical glass bulb containing the iodine was inserted in the path of the light from the arc. A number of the iron lines in the green are found to have iodine lines superimposed more or less centrally upon them. Plate V*b*, illustrating this, shows the line λ 5424 enlarged 6.7 fold, the upper end corresponding to the negative pole of the arc, the lower end to a point near the center. On the original plate three iodine absorption lines can be distinguished, one of which is seen in the reproduc-

tion to lie clearly upon the iron line. At the center of the arc it is obviously well to the violet side of the maximum of the line, while at the pole it is plainly upon the red side. Although the other two iodine lines fall upon the violet wing of the strongest part of the line, cutting it off sharply and greatly reducing the width of this

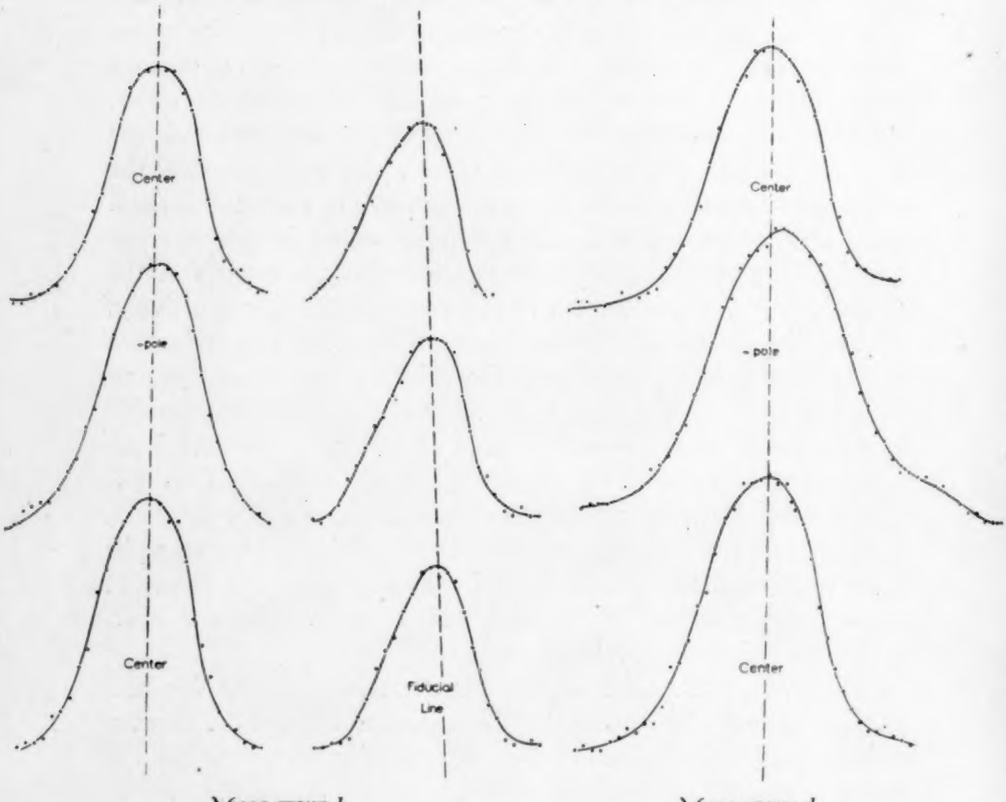
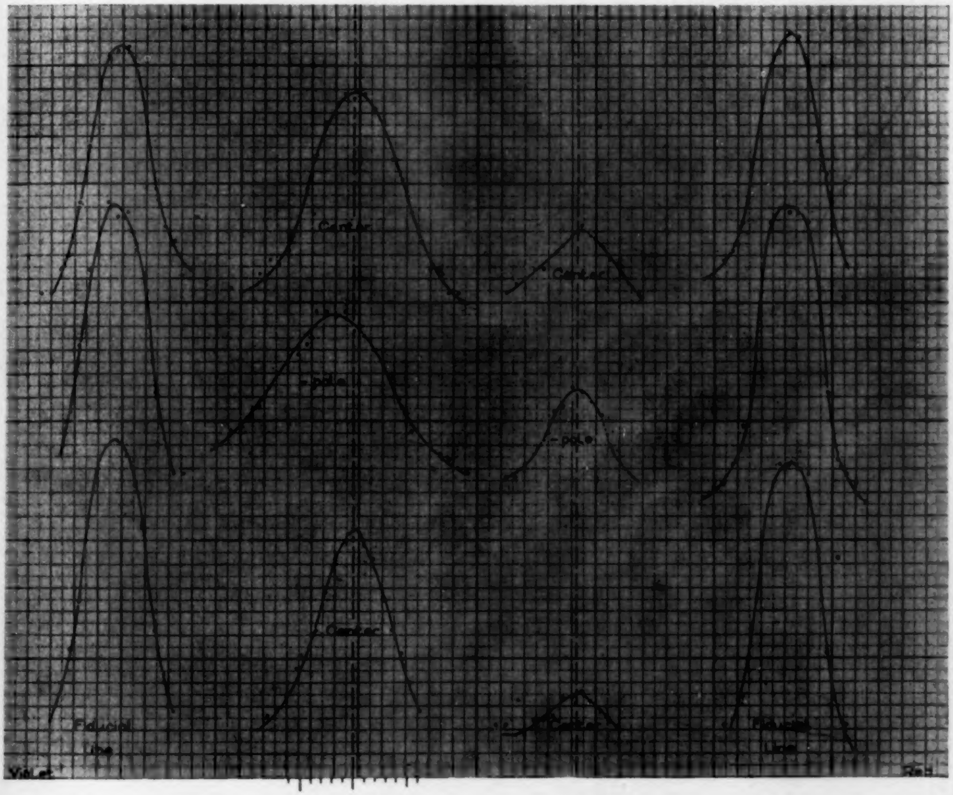


FIG. 2.—Microphotometer curves showing displacement at pole for λ 6400, group *d*, but not for λ 6393, group *b*.

portion, the shift of the maximum of the iron line between pole and center is clearly apparent. It appears to us that the only possible interpretation of this observation is that the wave-length corresponding to the maximum intensity of the iron line is less at the pole than at the center of the arc. On the original

negative a smaller shift in the same direction is shown at the positive pole.

The question whether the unsymmetrical broadening of spectrum lines may be accompanied by displacements is an old one.[1]



λ 5364.8 group *e* λ 5365.4 group *a*

FIG. 3.—Microphotometer curves showing displacement at pole for λ 5364.8, group *e*, but not for λ 5365.4, group *a*.

Professor Kayser remarks that the position of a spectrum line is determined by the intensity maximum. In measuring these lines,

[1] Kayser, *Handbuch der Spectroscopie*, **2**, 297, 1902; *Astrophysical Journal*, **26**, 191, 1907; Exner and Haschek, *Die Spektren der Elemente bei normalem Druck*, **1**; *Sitzungsberichte der Wiener Akad.*, **116**, Abt. II*a*, 323, 1907; Eder and Valenta, *Astrophysical Journal*, **19**, 251, 1904.

the effort has been to set upon the maximum; a double cross-hair in the observing microscope, with the space adapted to the width of the particular line under examination, has proved a satisfactory device for the purpose.

There are three points, each of which appears to us to establish the reality of the displacements of the maxima of the line between pole and center: (1) the persistence of the displacements when the intensity and width of the line at the pole are less than for the same line produced in the central section of the arc, since the maximum is the effective portion of the line when exposure times are shortened; (2) the shift of the intensity maximum when photometrically determined; (3) the relative position of the maxima at pole and center with respect to the superimposed iodine lines.

IV. PRESSURE IN THE ARC

The question whether the pressure differs from point to point in the arc sufficiently to account for these displacements was investigated by examining the behavior of lines of known pressure-shift that remain symmetrical under wide ranges of pressure and density. It is a comparatively easy matter to make differential measurements of high-dispersion spectra of good lines to the third decimal place, but a difficult matter to obtain comparison spectra that are reliable to this degree of precision. It was only after developing a method of obtaining rigorously simultaneous exposures that we were able to convince ourselves that no general pressure differences occur in the arc. The data in Table I illustrate the relative behavior of the lines of groups *a*, *b*, *c5*, *d*, and *e*. At the bottom are appended the mean pressure displacements per atmosphere for the same groups of lines.

The displacements of the lines of groups *a* and *b* do not exceed the limit of error, but the pressure displacement per atmosphere, in the case of the *b* lines, is of a magnitude to show a change of pressure of one-tenth of an atmosphere, while to produce the displacements of the lines of groups *c5* and *d* an increase of pressure of one and a half to two atmospheres would be required. The behavior of the lines of group *e*, on the other hand, would indicate a great reduction of pressure. It appears therefore from the data

that the displacements shown by the lines of groups *c5*, *d*, and *e* are not due to a *general* increase of pressure in the vapors near the pole of the arc.

TABLE I

NEGATIVE POLE *minus* CENTER OF THE ARC

Group *a*		Group *b*		Group *c5*		Group *d*		Group *e*	
λ	P−C	λ	P−C	λ	P−C	λ	P−C	λ	P−C
5328	0.0000 A	6136	+0.0007 A	4890	+0.014 A	5281	+0.019 A	5364	−0.028 A
5332	−0.0002	6137	−0.0004	4919	+0.016	5283	+0.018	5367	−0.025
5341	−0.0006	6213	+0.0003	4920	+0.013	5324	+0.015	5369	−0.020
5497	−0.0003	6219	+0.0010	4938	+0.018	5339	+0.019	5383	−0.025
5501	−0.0004	6230	+0.0004	4957	+0.014	5393	+0.017	5404	−0.025
5506	+0.0004	6252	0.0000	4957	+0.014	5653	+0.016	5410	−0.026
Mean.	−0.0002 A		+0.0003 A		+0.015 A		+0.018 A		−0.025 A
Displ. per atm.	+0.0036		+0.0094		+0.0094		+0.0092		+0.0017

V. OTHER CONDITIONS IN THE ARC

Density.—The absence of a general increase in pressure between the center and the negative pole led us at first to consider density as a possible cause of the pole effect. The concentration of luminosity is marked at the poles and is quantitatively indicated by the longer exposure upon the center of the arc required to equalize the intensities of the pole and center spectra. To test the possible effect of density upon the position of these sensitive lines, a pair of furnace plates was taken for us by Mr. King with 0.2 g and 2.0 g of iron, respectively. The iron deposit upon the walls of the graphite tubes showed that in both cases all of the iron was vaporized, so that the ratio of the vapor densities may be considered comparable to the quantities of metallic iron. Neighboring lines belonging to group *a*, the flame lines, were used as standards. Their wave-lengths are independent of the current-density and the part of the arc serving as a source, and King has found[1] that for lines of this type the pressure displacement does not depend upon the density of the furnace vapor. Our measurements are given in

[1] *Mt. Wilson Contr.*, No. 60, pp. 21–27; *Astrophysical Journal*, **35**, 203–209, 1912.

Table II, and show no increase of wave-length with the increased density, though for the same lines the increase in passing from the center to the pole of the arc is 0.0195 A. Some lines of manganese present as an impurity in the iron poles of the arc show displacements at the negative pole of the same order as the neighboring lines of iron. It does not seem probable that the density of a trace of vapor would be very appreciable even at the negative pole.

TABLE II

LINES OF GROUP *d* UNDER VARYING DENSITY AND TEMPERATURE IN THE FURNACE

λ	Density $\lambda_{HD}-\lambda_{LD}$	Pole Effect	λ	Temperature $\lambda_{HT}-\lambda_{LT}$	Pole Effect
5266	0.000 A	+0.017 A	5232	0.000 A	+0.025 A
5324	−0.004	+0.015	5266	0.000	+0.017
5586	+0.001	+0.024	5324	−0.001	+0.015
5615	−0.003	+0.022			
Means	−0.0015	+0.0195		−0.0003	+0.019

Temperature.—The effect of a variation in temperature was investigated by comparing the wave-lengths of these sensitive lines at temperatures in the furnace as widely different as practicable (2100°–2600° C.). They are high-temperature lines, and hence appear strongly only at the highest furnace temperatures, but the three given in Table II are measurable at the lower temperature and are free from the lines of the carbon flutings appearing so abundantly at the highest temperature. With the neighboring lines of group *a* as standards, no increase in wave-length was found for a 25 per cent increase in temperature, though between the center and the negative pole of the arc the displacement is 0.019 A.

In vacuo.—In order to make a preliminary test of the probable effect of electrical conditions, a 6 mm arc of the same form as that used under normal pressure and actuated by a current of like intensity was operated in a vacuum chamber at pressures of 0.5 cm and 10 cm of mercury; comparisons were made between the negative pole and center with practical equality of intensities for the lines under examination. In Table III data are given showing the results of measurement for five groups of lines.

For the lines of groups *a* and *b* the measurements show the same results as at atmospheric pressure, that is, no pole effect in either case; but for these typical lines of groups *c*5, *d*, and *e* the pole effects disappear with pressures below 10 cm of mercury, though at atmospheric pressure they are +0.012, +0.017, and −0.027 A, respectively.

TABLE III

NEGATIVE POLE *minus* CENTER *in vacuo*

λ	Group *a*	λ	Group *b*	λ	Group *c*5	λ	Group *d*	λ	Group *e*
5270	+0.0010	4547	0.0000	4607	+0.0008	5266	−0.0006	5367	−0.0005
5328	−0.0005	4592	+0.0005	4611	+0.0008	5281	+0.0005	5383	−0.0005
5332	+0.0002	4602	+0.0002	4736	0.0000	5283	−0.0006	5410	0.0000
5341	0.0000	4786	0.0000	4859	−0.0005	5302	−0.0004	5415	0.0000
5397	−0.0003	4789	−0.0010	4872	−0.0002	5324	+0.0018	5424	0.0000
Means	+0.0001		−0.0001		+0.0002		+0.0001		−0.0001

Though the arc *in vacuo* was of the same type and length and carried the same current as the arc under normal pressure, its appearance was strikingly different; the luminosity in such a case is more evenly diffused over the poles, and does not issue from a point source. Though the polar region is brighter than the central zone, the concentration in the core is practically absent. The disappearance of the pole effect under these conditions indicates that the potential difference plays a minor rôle, if any, but a more definitive investigation is to be undertaken.

Relation to luminosity.—The observations of Fabry and Buisson[1] brought out a striking difference of luminosity in the positive and negative regions of the iron arc, a difference confirmed by the change in the relative exposure times required to equalize the intensities of pole and center spectra on our plates. The ratio of center to pole exposure is greater for the red than for the violet at the negative pole. There is, therefore, a greater apparent change in radiation conditions in passing from the center to the negative pole in red than in violet light, but the loss of light through scattering and absorption in the outer layers of the arc vapor is undoubt-

[1] *Journal de physique* (4), **9**, 929, 1910.

edly greater for shorter wave-lengths, and as the negative pole is surrounded by a much thicker envelope of cooler vapor than is the central portion of the arc, the apparent relative luminosity for these parts of the arc should be quite different for widely separated spectral regions. At the positive pole the greatest apparent change in intensity of radiation between center and pole is in violet light. If the pole effect is closely related to these conditions, one would expect the displacements for the longer wave-lengths to be greater at the negative than at the positive pole, and for the shorter wave-lengths the reverse relation to obtain. The observations are given in Table IV.

TABLE IV

COMPARATIVE DISPLACEMENTS AT POSITIVE AND NEGATIVE POLES

Group	Ultra-Violet			Green		
	λ	Neg. Pole −Center	Pos. Pole −Center	λ	Neg. Pole −Center	Pos. Pole −Center
d	3322.50	+0.010	−0.001	5339.94	+0.019	+0.004
	3407.47	+0.006	0.000	5393.19	+0.017	+0.004
	3426.65	+0.004	+0.002	5569.63	+0.020	+0.003
	3667.28	+0.008	0.000	5602.96	+0.019	+0.007
Means		+0.007	0.000		+0.019	+0.004
e	3689.46	−0.011	−0.004	5367.46	−0.025	−0.005
	3694.00	−0.014	−0.004	5369.96	−0.020	−0.004
	3748.96	−0.014	−0.006	5410.90	−0.026	−0.004
	3754.50	−0.008	−0.006	5415.19	−0.030	−0.006
Means		−0.012	−0.005		−0.025	−0.005

For the region of λ 5400 the displacements are largest at the negative pole, but the reverse does not hold for the ultra-violet, though for the lines of group *e* the difference between the two poles is less for the shorter wave-lengths. The evidence against a relationship between luminosity and pole effect is strong in the case of the lines of group *d*, which appear to constitute a homogeneous group, since their pressure-shift varies as the cube of the wave-length; but the lines in the two sections of group *e* are not so connected and upon other grounds seem to be less intimately related. Under the reduced pressures used the pole effect disappears for the

lines near λ 5400, but persists for the lines in the violet. The true pressure-shift for the lines in the green is very near zero, while for the violet region these lines show displacements to the violet under increase of pressure. A more complete consideration of this class of lines will appear in a later paper.

Local pressure.—In comparing the spectra of the core and the flame of the arc, Adams[1] found that the ratio of core to flame intensity is highest for the lines of groups *d* and *e*, and lowest for lines of group *a*. Not only are the lines under consideration relatively the strongest in the core when compared to the flame of the arc, but they also increase in intensity more rapidly on approaching the poles than any other group of lines, so that the energy density for them is high in the limited volume of vapor in which they originate. This raises the question of a local increase in pressure in the core of the arc near the pole, which might produce an effect on those lines of which the core is mainly the source, while in the case of lines of groups *a* and *b*, for which the core plays a less important rôle, the effect may escape observation. In general the measurements for such lines show a slight indication of displacement, but of a magnitude not considered to be within the range of precision of our measurements. A possible method of approaching the question of a local increase in pressure is to compare the pole effect with the pressure displacements for the same lines. We have measured the pressure-shift per atmosphere by comparing the center of the 6 mm arc under pressures of 0.5 cm and 10.0 cm with the center of a duplicate arc under normal pressure. Upon this point St. John and Ware say:

> Neither the small pressure-changes of about one-fifth of an atmosphere taken advantage of in this investigation, nor the high pressures used by Gale and Adams are well adapted to the study of lines of this type, and it is purposed to examine *in vacuo* and under normal pressure the behavior of an extended list of lines belonging to groups *d* and *e*.[2]

The precision which our preliminary measurements give justifies the opinion implied in the reference made. Taken in the central section of the 6 mm arc in both cases, the lines are of good

[1] *Mt. Wilson Contr.*, No. 40; *Astrophysical Journal*, **30**, 86, 1909.

[2] *Mt. Wilson Contr.*, No. 61, p. 21; *Astrophysical Journal*, **36**, 37, 1912.

quality, and the displacements may be determined with a surprising degree of accuracy, but the results show that the pressure displacements formerly attributed to these sensitive lines are greatly in error through pole effect.

TABLE V

PRESSURE-SHIFT, POLE EFFECT, AND WAVE-LENGTH

Group	No. of Lines	Mean λ	Δ per Atm.	Pole Effect	Pressure-Increase
d......	25	4085	+0.0048 A	+0.0099 A	2.1 atm.
d......	12	5528	+0.0089	+0.0206	2.3
d......	6	6350	+0.0160	+0.0185	1.2
c5	16	4766	+0.0093	+0.0119	1.3
e......	7	3755	−0.0035	−0.009	2.6
e......	8	5392	+0.0017	−0.026	?

The results for representative groups of lines are assembled in Table V. The increases of pressure on the assumption that the effect is due to pressure at the negative pole, given in the last column, show variations exceeding the limits of error.

The pressure displacements for the three sections of group *d*, shown in Table V, are related as the cube of the wave-length, confirming the conclusion of Gale and Adams.[1] The values calculated from the weighted equation

$$\Delta\lambda=\left(\frac{\lambda}{5000}\right)^3 0.00804$$

show residuals as in Table VI.

TABLE VI

AGREEMENT WITH CUBE LAW

MEAN λ	Δλ			WEIGHT
	Observed	Calculated	Obs.−Calc.	
4085......	0.0048 A	0.0043 A	+0.0005 A	25
5528......	0.0089	0.0108	−0.0019	12
6350......	0.0160	0.0164	−0.0004	6

[1] *Mt. Wilson Contr.*, No. 58, pp. 22–26; *Astrophysical Journal*, **35**, 32–36, 1912.

On taking account of the weights, it will be seen that the small deviations of the observed points from the theoretical curve are properly distributed; but an examination of the pole effects in Table V shows that they do not follow the cube law even approximately. Since the pole effects are not so related to the wave-length, it is evident that pressure alone does not explain them.

VI. IDENTIFICATION AND DISTRIBUTION OF AFFECTED LINES

Between λ 2979 and λ 6678 we have examined 1570 lines; when the negative pole is compared with the center, we find 286 lines showing displacements to the red and 80 with displacements to the violet; that is, 23 per cent of the lines are affected. In Table VII the lines showing displacements to the red are listed, and in Table VIII those which are displaced to the violet. The first column identifies the lines by their wave-lengths in international units to the second decimal place. The present available determinations of wave-length show variations of such magnitude for the majority of the lines listed that it seems advisable for the present to omit the third place. The second column gives the intensity and character according to Burns[1] for the lines contained in his tables; the others are in general very weak lines. The displacements, negative pole minus center, are in the third column; when followed by I.S., the corresponding lines are international standards of the second order. No attempt has been made to separate the groups *c5* and *d*, nor to indicate any subgroups. This is best done by means of the pressure-shifts, which are at present under investigation.

The distribution is shown in Table IX. The regions are so selected that the number of affected lines per 100 A is fairly uniform in each. From the point of view of one who uses the iron lines as standards of wave-length, as reference lines in solar or stellar investigations, or as a basis for intensity comparisons, not only is the absolute distribution of interest, but also the proportion of affected lines in a given spectral region. The sensitive lines are numerous in sections of the ultra-violet; they form, however, a small percentage of the total number of lines there, but in the

[1] *Lick Observatory Bulletin*, **8**, 27, 1913.

TABLE VII

FE LINES, GROUPS *c5* AND *d*

NEGATIVE POLE *minus* CENTER OF ARC

DISPLACEMENTS TO LONGER WAVE-LENGTHS

λ (Int. Units)	Burns	P−C	λ (Int. Units)	Burns	P−C
2991.65	4b	+0.006 A	3739.54	1h	+0.014 A
3012.46	2b	+0.006	3740.06	1	+0.019
3048.47	2H	+0.007	3787.60		+0.007
3093.89	2b	+0.006	3789.44	1	+0.015
3154.51	2b	+0.007	3811.01		+0.010
3188.59	4b	+0.004	3814.79		+0.012
3208.48	4	+0.004	3817.65		+0.030
3209.33	4b	+0.010	3830.87	1	+0.008
3211.69	4b	+0.004	3920.85	1b	+0.008
3322.50	4b	+0.010	3928.09	1h	+0.010
3407.47	7l	+0.006	3941.29	2b	+0.014
3410.90	1	+0.006	3947.00	2b	+0.004
3426.65	6	+0.004	3948.11	3b	+0.009
3438.51	3b	+0.004	3955.36	2b	+0.020
3445.78	2b	+0.007	3957.03	2b	+0.018
3459.74	1b	+0.004	3963.11	2b	+0.012
3474.44	2	+0.007	3965.44	1	+0.010
3518.68		+0.011	3976.62	2	+0.004
3522.27		+0.004	4018.28	2b	+0.004
3532.56		+0.012	4024.75	2	+0.013
3568.98	4	+0.010	4030.51	3b	+0.011
3583.67		+0.013	4058.23	2b	+0.006
3586.61		+0.010	4065.40	1	+0.006
3587.25	2H	+0.010	4072.52	1b	+0.008
3592.66		+0.010	4073.76		+0.008
3599.13		+0.012	4083.78	1	+0.008
3604.69		+0.007	4084.51	4	+0.007
3607.55		+0.008	4101.27	1b	+0.008
3612.08	4	+0.004	4104.13	2b	+0.008
3613.47		+0.007	4109.07	1	+0.004
3616.32		+0.009	4112.98	2b	+0.020
3620.47		+0.007	4118.90	1	+0.010
3635.20		+0.010	4125.63	1	+0.017
3636.49		+0.007	4133.87	2b	+0.009
3643.14		+0.009	4150.28	2b	+0.018
3644.81		+0.009	4153.92	4b	+0.012
3655.68		+0.016	4154.82	3	+0.004
3662.85		+0.014	4157.80	3b	+0.018
3664.56	2	+0.004	4158.81	2b	+0.014
3665.81		+0.006	4171.70	2	+0.007
3666.26		+0.011	4187.05	6	+0.008
3667.28	4h	+0.008	4187.81	6	+0.010
3667.99	2h	+0.004	4191.44	6	+0.010 I.S.
3676.88	1b	+0.006	4195.34	3b	+0.009
3688.48	1b	+0.005	4196.22	2	+0.011
3697.44	2h	+0.006	4198.31	6	+0.010
3703.70	1h	+0.005	4198.64	2	+0.009
3721.51	1	+0.005	4210.36	6	+0.006
3735.33	3	+0.005	4217.56	2b	+0.008

TABLE VII—*Continued*

λ (Int. Units)	Burns	P−C	λ (Int. Units)	Burns	P−C
4222.22	5	+0.006 A	4938.83	5	+0.018 A
4225.46	4b	+0.004	4944.34		+0.024
4227.45	7	+0.014	4946.40	2	+0.024
4233.16	1	+0.007	4950.12	1	+0.020
4233.61	6	+0.010 I.S.	4954.11		+0.016
4235.95	8	+0.009	4954.41		+0.029
4238.83	4b	+0.011	4957.31	5	+0.014
4247.44	5b	+0.011	4957.61	10	+0.014
4250.13	7	+0.010	4960.65		+0.020
4260.49	10	+0.012	4966.10	5	+0.017 I.S.
4264.21	2	+0.006	4969.94	2b	+0.045
4271.17	7	+0.010	4970.80		+0.026
4299.25	7	+0.009	4973.11	2	+0.018
4343.28	2	+0.012	4978.61	2	+0.018
4401.30	3	+0.004	4982.52	4b	+0.026
4407.71	2	+0.011	4983.27	3b	+0.020
4446.85	2b	+0.012	4985.27	3	+0.016
4462.01	3b	+0.020	4985.56	3	+0.019
4469.39	4b	+0.008	4988.97	2	+0.018
4531.64	2	+0.006	4991.29		+0.030
4581.52	2	+0.012	5001.88	5	+0.023
4598.13	2b	+0.012	5002.82		+0.022
4607.66	4	+0.014	5005.73	4	+0.032
4611.29	4b	+0.006	5006.13	5	+0.016
4613.23	3	+0.011	5007.31	2h	+0.030
4625.06	4	+0.012	5014.96	4	+0.022
4637.52	4	+0.012	5022.25	4	+0.024
4654.64	3b	+0.007	5027.14	2h	+0.014
4668.15	4	+0.013	5039.27	2b	+0.026
4707.29	5	+0.012 I.S.	5044.22		+0.018
4709.09	2	+0.011	5048.45	2	+0.021
4727.42	2	+0.024	5068.78	4	+0.030
4736.79	5	+0.012 I.S.	5073.67		+0.035
4754.05	5	+0.010 Mn	5076.28		+0.022
4783.44	4	+0.012 Mn	5090.79	3h	+0.010
4823.53	4	+0.011 Mn	5099.05		+0.020
4859.76	5	+0.014 I.S.	5125.14	2b	+0.020
4871.33	8	+0.010	5126.21		+0.020
4872.15	8	+0.014	5136.06		+0.028
4878.22	6	+0.012 I.S.	5137.39	3	+0.014
4890.77	7	+0.014	5139.27	6	+0.021
4891.50	9	+0.012	5139.48	8	+0.020
4903.32	5	+0.009 I.S.	5162.32	5b	+0.030
4915.60	1	+0.018	5165.43	2b	+0.010
4917.20		+0.012	5191.47	7	+0.019
4919.01	8	+0.016 I.S.	5192.36	8	+0.019 I.S.
4920.52	10	+0.013	5208.61	4	+0.020
4922.38		+0.014	5215.20	4	+0.020
4923.94		+0.046	5217.41	4	+0.016
4930.33	1	+0.015	5226.88	5	+0.023
4932.21		+0.021	5229.52		+0.019
4933.64		+0.045	5229.86		+0.015
4934.02	1h	+0.005	5232.96	8	+0.025 I.S.
4938.18	1	+0.018	5236.19	1	+0.020

TABLE VII—*Continued*

λ (Int. Units)	Burns	P−C	λ (Int. Units)	Burns	P−C
5263.32	5	+0.020 A	5655.51	2	+0.016 A
5266.04		+0.012	5658.84	4	+0.023 I.S.
5266.57	8	+0.017 I.S.	5662.53	3	+0.018
5273.18	3	+0.014	5701.47	4	+0.010
5281.80	5	+0.019	5705.48	1	+0.031
5283.64	7	+0.018	5709.40	3	+0.024
5302.32	5	+0.014 I.S.	5711.87	2	+0.010
5324.19	6	+0.015 I.S.	5712.15	2	+0.017
5339.94	3	+0.019	5715.11	1	+0.007
5353.38	2	+0.014	5717.85	3	+0.018
5389.46	2	+0.015	5731.78	3	+0.010
5391.49	1	+0.028	5753.14	3	+0.028
5393.19	4	+0.017	5763.02	4	+0.028
5466.42	3	+0.020	5775.10	3	+0.025
5472.72	1	+0.022	5782.15	1	+0.020
5473.91	3	+0.014	5791.04	2	+0.020
5476.58	4	+0.019	5809.25	2	+0.041
5480.87	2	+0.011	5859.61	3b	+0.025
5487.78	3	+0.036	5883.84	3	+0.024
5522.46	2	+0.013	5905.68	2	+0.007
5525.55	2	+0.010	5934.68	4	+0.028
5543.94	2	+0.018	5952.75	4	+0.021
5560.23	1	+0.025	5976.80	2	+0.029
5563.61	3	+0.021	5983.71	2h	+0.018
5567.40	2	+0.016	6003.04	3	+0.020
5569.63	5	+0.020 I.S.	6008.58	3	+0.024
5572.86	5	+0.025	6013.52	2	+0.019 Mn
5576.10	4	+0.021	6016.66	2	+0.020 Mn
5586.77	6	+0.024 I.S.	6021.82	2h	+0.018 Mn
5600.24	1	+0.018	6141.13		+0.016
5602.79	2	+0.016	6180.22	2	+0.011
5602.96	3	+0.019	6232.67	2	+0.017
5608.16		+0.021	6246.34	4	+0.020
5615.66	6	+0.022 I.S.	6301.52	5	+0.016
5618.65	1	+0.010	6302.51	3	+0.015
5624.56	5	+0.023	6336.84	4	+0.020
5633.97	2	+0.023	6400.02	5	+0.019
5638.28	3	+0.015	6408.04	4	+0.017
5641.46	2	+0.025	6411.67	5	+0.019
5655.18		+0.014	6419.99	5	+0.024

region λ 4900–λ 5050, for example, there are 34 affected lines. Burns gives for this region 24 Fe lines of intensity 2 or stronger; of these 20 are sensitive to the pole effect. From λ 5500 to λ 6000 they are practically the only lines in the iron spectrum. The lines giving displacements to the violet at the negative pole have a stronger gregarious tendency showing in the ultra-violet and to the red of λ 5364. When the ultimate groups of the iron lines are

once determined from their reactions to various physical conditions, it may be possible to find definite series relationships even in so complex a system as the iron spectrum.

TABLE VIII

FE LINES, GROUP *e*

NEGATIVE POLE *minus* CENTER OF ARC

DISPLACEMENTS TO SHORTER WAVE-LENGTH

λ (Int. Units)	Burns	P−C	λ (Int. Units)	Burns	P−C
3157.88	4	−0.004 A	5074.75	2b	−0.022 A
3160.65	6	−0.004	5079.00		−0.013
3205.40	7b	−0.004	5096.99	3b	−0.016
3210.46	2	−0.006	5133.67	5b	−0.045
3244.19	1	−0.004	5153.20		−0.035
3251.24	5h	−0.004	5364.86	3h	−0.028
3516.41	3	−0.008	5367.46	3b	−0.025
3518.86		−0.006	5369.96	4b	−0.020
3529.82	4	−0.004	5383.37	5b	−0.025
3532.10		−0.004	5400.50	2b	−0.011
3533.00	4	−0.004	5404.13	3b	−0.025
3549.87	3	−0.010	5410.90	3b	−0.026
3582.58		−0.013	5415.19	4b	−0.030
3588.52		−0.006	5424.06	4b	−0.027
3594.63	5	−0.004	5432.96		−0.023
3604.28		−0.008	5445.04	2h	−0.020
3610.15	5h	−0.007	5462.96	2b	−0.014
3616.58	4h	−0.014	5463.27	4b	−0.016
3633.84	4h	−0.006	5543.18	2	−0.014
3634.68		−0.006	5554.88	3b	−0.019
3650.03	3h	−0.007	5565.78	3	−0.019
3682.21	1	−0.003	5594.66	2	−0.023
3689.90	1b	−0.011	5598.31	3	−0.025
3694.00	6	−0.014	5686.53	3	−0.028
3701.08	6	−0.009	5693.64	2	−0.007
3726.92	3h	−0.006	5705.99	2	−0.019
3744.09	2h	−0.011	5816.36	3	−0.026
3748.96	3h	−0.014	5862.35	4b	−0.018
3754.50	2b	−0.008	5914.16	6	−0.003
3773.69	2b	−0.012	5930.18	5	−0.020
3797.95	1	−0.012	5984.81	3	−0.022
3845.26	5	−0.007	5987.06	2	−0.016
3966.62	5b	−0.004	6007.96	2h	−0.016
4172.64		−0.017	6020.18	2l	−0.012
4200.92	2	−0.007	6024.06	4h	−0.015
4224.51	2b	−0.006	6042.08	2	−0.012
4433.22	2b	−0.006	6055.99	3h	−0.015
4960.93		−0.035	6078.48	3	−0.022
4967.89	2	−0.016	6102.18	3	−0.010
5065.02	3b	−0.029	6103.19	2h	−0.015

TABLE IX

DISTRIBUTION OF AFFECTED LINES

Displaced to Red			Displaced to Violet		
Region	Total	Per 100 A	Region	Total	Per 100 A
3000–3400...	10	2	3100–3300...	6	3
3400–3900...	47	9	3300–3500...	0	0
3900–4300...	54	14	3500–3800...	25	8
4300–4600...	9	3	3800–4500...	6	1
4600–4900...	20	7	4500–4900...	0	0
4900–5300...	72	18	4900–5200...	8	3
5300–5800...	50	10	5200–5300...	0	0
5800–6100...	13	4	5300–5600...	18	6
6100–6420...	11	3	5600–6200...	17	3
	286			80	

VII. WORKING CONDITIONS IN THE IRON ARC

Aside from the theoretical interest in the changes of wave-length considered in this paper, reference may be made to the following practical considerations:

1. A number of these sensitive lines are included among the international standards of the second order adopted by the International Union for Co-operation in Solar Research.

2. There are regions of the iron spectrum in which few or no other lines are available for standards; for example, from λ 4900 to λ 5050 and from λ 5500 to λ 6000.

3. In various laboratories there are in progress redeterminations, based upon the iron standards, of the wave-lengths in international units of the lines of many elements. In these redeterminations the instrument most commonly used is the concave grating in the usual Rowland mounting, and in practice the slit of the spectrograph is parallel to the axis of the arc and includes the major part of its length. The astigmatism under these conditions introduces more or less pole effect, and to that degree vitiates results involving lines of the character under consideration. The practice of reversing the current in the arc in order to overcome the tendency to produce wedge-shaped lines when the slit and the axis of the arc are parallel, obscures, but does not eliminate, the pole effect.

Since the redeterminations aim at a precision of 0.002 to 0.003 A, it is necessary to take the pole effect into consideration.

4. Lines of the type considered are not limited to iron, but are present in the spectra of other elements, the detailed investigation of which is necessary before safe deductions can be made from their use in astrophysical investigations, or before their wave-lengths can be determined with the requisite precision.

5. The arc lines are often used as a basis for intensity comparisons, and for such purposes reliable results depend upon employing suitable arc arrangements.

TABLE X

DISPLACEMENTS, POLE DISTANCE, AND CURRENT

	Neg. Pole and Center	1 mm from Neg. Pole and Center	Pos. Pole and Center	12−4 Amperes	7−5 Amperes
Group *a*.....	−0.0005 A	+0.0005 A	−0.0002 A	−0.0004 A	−0.0002 A
Group *d*.....	+0.021	+0.009	+0.003	+0.007	+0.001
Group *e*.....	−0.025	−0.014	−0.006	−0.012	−0.003

It is of importance then to determine the practical conditions under which these sensitive lines may be used and the limits of the precision obtainable. In furtherance of such a purpose we have made comparisons between the center of the arc and the positive pole, the negative pole, and a point 1 mm from the negative pole, using the Pfund arc 6 mm long, carrying a current of 6 amperes under a pressure of 110 volts; and also between the centers of arcs carrying 5 and 7 amperes, and between arcs carrying 4 and 12 amperes. The data are shown in Table X. For the *d* lines the current may vary between 5 and 7 amperes without introducing errors exceeding the desired precision, and it appears that the small changes in current obtaining in practice, when care is taken to hold it constant in the standard arc, are without measurable effect. A more insidious source of error is the introduction of the pole effect by using light from any part of the arc except the middle zone, as nearness to the negative pole is accompanied by easily measurable displacements. It is important to hold closely to the mid-point of the arc, and advisable to approach the positive rather than the negative pole if any considerable length of the arc is

to be employed. In our experience the highest precision and the most uniform results are obtained by keeping the slit normal to the axis at its mid-point in a greatly enlarged image of the arc. The necessary conditions for high precision are difficult to realize in the classical mounting of the concave grating without some arrangement for rotating the image of the arc. The ultra-violet plates for this investigation were taken with a concave grating in a Rowland mounting arranged in a vertical plane and with the slit normal to the axis of the arc.

The effects arising from differences in arc conditions and types of spectrographs employed are, as has been mentioned, manifest in the determinations of the tertiary standards by different observers. They are also apparent in other lines of work. We wished to compare our results for the pressure displacement of these lines with those taken under the widest range of pressure, and turned to Duffield's interesting paper.[1] He worked with pressure differences of 3 to 100 atmospheres, and used the ordinary mounting of the concave grating with the slit parallel to the axis of the arc, the slit and the image of the arc being of the same height. His procedure involved the pole effect in somewhat varying degrees, as he changed both the length of the arc and the current-strength, and at high pressures the exposures were made by a series of flashes, a process that intensifies the effect of the polar influence. From his Tables I and II the lines showing pole effects are selected, and the pressure-shifts per atmosphere, deduced from the pressure differences used, are given in Table XI. Unreversed lines only are considered and the means of his two sets are taken. In the last column are shown the displacements for one atmosphere found by us for some of the same lines of group *d* in passing from a vacuum to atmospheric pressure. Duffield's results vary from 0.033 A per atmosphere, determined from the pressure difference of 3 atmospheres, to 0.006 A, deduced from the pressure difference of 80 to 100 atmospheres. These discrepancies are explicable as pole effect. At low pressures a larger proportion of its influence would appear as an increment to the pressure-shift, while with increasing pressures the pole effect would play a decreasing rôle. For the lines of group *a*

[1] *Phil. Trans.*, A, **208**, 111, 1908.

no pole effect is shown by our measurements. There are in Duffield's tables two lines of this group. The pressure-shifts per atmosphere deduced from pressure differences of 10 and 80 atmospheres are 0.0022 A and 0.0015 A respectively, an agreement in striking contrast to the results for lines showing pole effect.

TABLE XI

PRESSURE-SHIFT PER ATMOSPHERE—DUFFIELD

λ	Atmospheres						
	3	5	10	80	95	100	St. John and Babcock
4299....	....	0.0128	....	0.0067	0.0060	0.0060	....
4236....	....	0.0126	0.0147	0.0083	0.0082	0.0086	0.0046
4233....	0.0330	0.0172	0.0163	0.0060	....	0.0064	0.0057
4227....	0.0377	0.0240	0.0125	....	....	....	0.0090
4222....	0.0287	0.0150	0.0151	....	....	....	....
4210....	0.0300	0.0134	0.0090	0.0034	0.0040	0.0048	0.0019
4191....	....	0.0162	0.0155	0.0057	0.00	0.0064	0.0027
4187....	0.0407	....	....	....	....	....	0.0032
4187....	0.0353	0.0152	0.0123	....	....	....	0.0038
Means ..	0.0326	0.0146	0.0135	0.0060	0.0061	0.0064	0.0044

The pole effect appears also in the interferometer determinations of λ atm. − λ *in vacuo* by Fabry and Buisson.[1] We have measured the values of λ atm. − λ *in vacuo* for the lines used by them, both with a grating spectrograph and with an interferometer. The respective results are given in the fourth and fifth columns of Table XII. The differences between the values of Fabry and Buisson and the means of our two determinations appear in the sixth column.

For the arc at atmospheric pressure Fabry and Buisson used a current of 3 amperes, while we used a current of 6 amperes. They say: "For the unsymmetrical lines the displacements would have been much greater if the intensity of the current had been stronger"; but the displacements observed by us are smaller than those found by them. A comparison of the differences in the sixth column with the pole effects in the seventh leaves no doubt that the large values obtained by them were mainly due to pole effect. As the maxima

[1] *Astrophysical Journal*, **31**, 112, 1910.

of the interference fringes correspond to the maxima of the emission lines, the measurements show actual displacements of the maxima under varying arc conditions, even when determined by the interferometer.

TABLE XII

λ ATM. *minus* λ *in vacuo* FOR SENSITIVE LINES

Group	λ	Fabry and Buisson	St. John and Babcock		Fabry and Buisson *minus* St. John and Babcock	Pole Effect
			Grating	Interferometer		
d..	4187.05	+0.011	+0.004	+0.004	+0.007	+0.008
	4191.44	+0.010	+0.003	−0.005	+0.006	+0.010
	4227.45	+0.020	+0.006	+0.009	+0.012	+0.014
	4233.61	+0.012	+0.006	+0.006	+0.006	+0.010
	4235.95	+0.011	+0.006	+0.005	+0.005	+0.009
	4250.13	+0.013	+0.005	+0.007	+0.007	+0.010
	4859.76	+0.017	+0.008	+0.005	+0.011	+0.014
	4871.33	+0.010	+0.013	+0.008	0.000	+0.010
e..	5415.19	−0.015	+0.001	....	−0.016	−0.025
	5424.06	−0.017	+0.001	0.000	−0.017	−0.026

There is also evidence that in the furnace spectra studied by King[1] the positions of these sensitive lines are affected by the phenomenon under consideration; for when their pressure displacements are compared with those of the stable lines in the same spectral region the furnace displacements exceed by 0.007 A those of the stable lines given under the same conditions, while those obtained from our arc determinations exceed the mean given by the stable lines under like conditions by 0.0026 A. Moreover, as shown in Table XIII, a striking agreement appears between the differences, furnace minus arc, and the pole effects given in the fourth and fifth columns respectively.

So much emphasis has been placed upon the differences between various determinations of pressure-shift that it may be well to call attention to the fact that the lines showing marked discrepancies are those catalogued in our Tables VII and VIII, and that among the iron lines examined by us some 1200 are free from pole effect and will normally yield definite values for pressure-shift. For example, in Table XIV we compare our interferometer determina-

[1] *Mt. Wilson Contr.*, No. 53; *Astrophysical Journal*, **34**, 37, 1911.

tions with the pressure-shift per atmosphere deduced from the displacements for 8 atmospheres found by Gale and Adams for lines of group *a*. Those lines of group *b* which we have measured in this way also show differences smaller than the errors of observation.

TABLE XIII

DISPLACEMENTS PER ATMOSPHERE IN FURNACE AND ARC
FE LINES, GROUP *d*

λ	Furnace	Arc	F−A	Pole Effect
4187.05........	+0.012	+0.0038	+0.008	+0.008 A
4187.81........	+0.013	+0.0033	+0.010	+0.010
4191.44........	+0.013	+0.0027	+0.010	+0.010
4198.31........	+0.014	+0.0113	+0.002	+0.010
4210.36........	+0.013	+0.0019	+0.011	+0.006
Mean...........	+0.013	+0.0046	+0.008	+0.009 A

The agreement for lines of this type, shown in Table XIV, is in strong contrast to the consistent difference for lines of group *d*. For the lines given in Table XII, the mean pressure-shift per atmosphere deduced from the displacement for 8 atmospheres found by Gale and Adams is +0.011 A, while, from our measurements based upon the lines as produced in the central zones of 6 mm, 6 ampere arcs, its mean value is +0.006 A, the difference being referable to the pole effect introduced by the short arcs used by them.

TABLE XIV

DEFINITE PRESSURE-SHIFT FOR STABLE LINES
GROUP *a*

	Gale and Adams	St. John and Babcock
4376................	+0.0022	+0.0020
5371................	+0.0036	+0.0031
5397................	+0.0036	+0.0029
5406................	+0.0034	+0.0045
5429................	+0.0036	+0.0033
5447................	+0.0039	+0.0030
5455................	+0.0036	+0.0035
	+0.0034	+0.0032

A remarkable agreement appears between the pressure displacements found for this group of sensitive lines by Fabry and Buisson (arc and interferometer), Gale and Adams (arc and grating), and King (furnace and grating), namely, 0.013, 0.011, and 0.013 A, respectively, while our measurements give only 0.006 A. It seems probable that some common factor was effective in producing these larger values, and a comparison of the excesses with the pole effect points to it as the operating cause.

It is a matter to be considered, whether the differences in wave-length between the center of the arc *in vacuo* and in air, as we have used it, are wholly due to pressure, or are still measurably influenced by pole effect. One way of approaching the question is to compare our results with those given by Duffield's data. As our value for the pressure-shift is 0.0044 A, and that deduced from the pressure differences of 100 atmospheres used by him is 0.006 A, it appears that under the conditions of our arrangement we are approaching closely, if we have not actually reached, the true pressure-shifts for lines subject to pole effect. This is further indicated by their varying as the cube of the wave-length, a relation shown by Gale and Adams to hold for the iron lines of groups *a* and *b*, which are free from pole effect. If the displacements found by us are complicated by the presence of another effect, not following the cube law, such a close agreement between observed and calculated values would be a remarkable coincidence. Another reason for thinking the 6-ampere arc, 6–7 mm in length, is practically free from the pole effect in the central plane is found in the way the displacements vary with current. Combining with ours some of Royds's[1] data for the differences between the centers of arcs carrying different currents, we find for the following current-changes the corresponding increases in wave-length:

5	to	7	amperes	+0.001 A
4.5	"	9.5	"	+0.003
4	"	12	"	+0.007

[1] During the progress of this investigation Dr. Royds's interesting paper appeared (*Bulletin* No. 40, Kodaikanal Observatory). The two investigations have proceeded along some common lines, and where the same ground is covered by the observations they are mutually confirmatory.

These indicate approximately displacements of 0.001, 0.002, and 0.004 A for 2-ampere increments to currents of 5, 7, and 10 amperes, respectively, and that decrease of current below 6 amperes is not necessary for the elimination of pole effect in the iron arc employed.

VIII. DISCUSSION

Though our observations show that displacements of the maximum intensity of certain types of lines occur between the center and pole of the iron arc, the cause of the displacements is not evident, nor is the mechanism plain that produces the unsymmetrical broadening which characterizes lines of these types, and with which the displacements are more closely related than with the pressure-shifts. Dr. Goos[1] attributed such displacements to differences of pressure in the arc. We do not find pressure differences of the order necessary to produce them. It may be said that probably all data relative to the pressure displacements of these sensitive lines are more or less affected by pole effect, and cannot serve for the accurate determination of pressure differences. Dr. Royds[2] considers density the predominating influence in producing the displacements. The fact that a tenfold change in vapor density in the furnace is without effect upon the position of the maximum is opposed to the density hypothesis, but the conditions obtaining in the furnace and in the arc are not strictly comparable. Exner and Haschek[3] suggest variability in the intensity of the components of a complex line under varying excitation as an explanation of displacement of the maximum. This would mean that in 18 per cent of the lines examined by us a close-lying satellite to the red of the principal component given by the central zone of the arc increased in relative intensity on approaching the negative pole, and that in 5 per cent of the lines the component of increased relative intensity was to the violet. More observational data than are now available are necessary to determine the probability of such a behavior of complex lines. The possibility of the occurrence of such variability is evident, but the observations of Nutting do not

[1] *Loc. cit.* [2] *Loc. cit.*

[3] *Sitzungsberichte Wiener Akad.*, **116**, Abt. II*a*, 323, 1907.

indicate it in the case of iron.[1] Neither of these points of view seems to us to offer a satisfactory explanation of the phenomenon, nor until definite results are obtained from investigations now in progress and to be undertaken do we wish to offer any further suggestion, as we are inclined to sympathize with Nutting when he says: "In conclusion, I wish to enter a plea for a simpler and broader basis for spectroscopy and a basis as free as possible from either assumption or speculation."[2]

Though we have not satisfied ourselves as to the explanation of the main phenomenon considered in this investigation, we feel that the data should be accessible to other investigators, particularly in view of the employment of the iron arc as a standard in the redeterminations in progress, and as a basis of sun and arc comparisons in the discussions bearing upon pressure, motion, anomalous dispersion, and Einstein effect in the solar atmosphere, as it is evident that the iron lines given in Tables VII and VIII, and lines of other elements behaving in a similar way, must be given separate consideration, and conclusions based upon them accepted with great caution.

We have suggested the term "pole effect," not only as a convenient designation of the phenomenon, but also as an indication of its dependence upon nearness to the pole. The effects due to increase of current appear to us a projection of the polar influence to a greater distance from the pole, while a shortening of the arc simply brings the central zone nearer to the pole.

We take this opportunity to express our appreciation of the assistance given us by various members of the staff; we are under particular obligation to Miss Ware for her able and unwearying help in the difficult measurements.

IX. SUMMARY

1. A combination of totally reflecting prisms and a rotating sector furnishes a means of making rigorously simultaneous exposures upon different sources.

[1] *Astrophysical Journal*, **22**, 7, 1906.

[2] *Astrophysical Journal*, **28**, 70, 1908.

2. Displacements of the maxima of certain unsymmetrical iron lines in passing from the center to the negative pole of the arc are shown by the persistence of the displacements when the widths of the lines at the pole are less than at the center of the arc, by the shift of the intensity maxima of the photometric curves, and by the relative position of the maxima at pole and center with respect to the superimposed iodine absorption lines.

3. Observations upon symmetrical lines with large pressure-shifts do not show a general increase of pressure in passing from the center to the negative pole sufficient to produce the observed displacements.

4. The wave-lengths of these sensitive lines are not affected by a tenfold change in the density of the iron vapor in the furnace.

5. Their wave-lengths are independent of a change in furnace temperature over the range of our observation, 2100°–2600° C.

6. Except in a very few special cases, the pole effect disappears *in vacuo* and in so far appears independent of electrical conditions.

7. It does not appear to be intimately related to the differences in luminosity between the positive and negative poles.

8. The variation of pole effect with wave-length does not follow the same law as pressure displacements.

9. Therefore an increase of pressure localized at the pole and in the core of the arc, where a greater proportional contribution is made to the total intensity of these lines than to any other groups, does not alone explain the displacements.

10. Between λ 2979 and λ 6678, of 1570 lines examined 286 show displacements to the red and 80 to the violet.

11. The affected lines are not distributed with any degree of uniformity, but show rather a tendency to cluster in certain regions.

12. In extensive regions of the spectrum, λ 4900–λ 5050, λ 5500–λ 6000, nearly all the lines are of this character.

13. An investigation of the limits of the working conditions in the 6-ampere, 110-volt, 6 mm iron arc of the Pfund form shows that the small fluctuations in current occurring in practice may be neglected, but that even when the arc is running steadily only a narrow equatorial zone is practically undisturbed by the pole effect.

14. Our study of the behavior of iron lines under different conditions furnishes additional ground for regarding the classification suggested by Gale and Adams as resting upon a real physical basis.

15. Emphasis is placed upon the necessity of considering the pole effect when the arc is used in comparisons of intensity, in redetermination of wave-lengths, and in astrophysical investigations.

MOUNT WILSON SOLAR OBSERVATORY
May 1915

STELLAR PARALLAX WORK AT THE McCORMICK OBSERVATORY

By S. A. MITCHELL

It has seemed advisable at the present time to publish the first measures of stellar parallaxes obtained by photography at the University of Virginia. It was generally conceded by the Clarks, while they were alive, that the 26-inch refractor was one of the best that they had ever made. Certainly the definition is excellent, with an almost utter absence of stray light.

Photographs were made with a yellow color-filter and Cramer isochromatic plates. The star images are small and clean-cut and lend themselves to accurate measurement, as the following results show. The plates, 5×7 inches, were measured on the Repsold measuring machine, which was kindly loaned by Columbia University. The methods of exposure, measurement, and reduction were substantially the same as those explained in this *Journal* by Schlesinger in Vols. **32**, **33**, and **34**, and by Slocum and Mitchell in **38**, 1, 1913.

70 Ophiuchi (18^h0^m, $+2°31'$)

This system has a very large proper motion, $1''.13$ per year. It has a period of 88 years, and has completed one revolution since its discovery. This star was put on the parallax program because it was on the Yerkes program,[1] and because it was desirable to see how accurately measures could be made on such a pair. The components are of magnitudes 4.3 and 6.0, and the present distance about $4''$. As the scale of the photographs is 1 mm $= 20''.8$, the stars were separated by 0.2 mm. A rotating sector was used to cut down the brightness of the two stars. The sector was opened so that the fainter star was nearly of the same brightness as the comparison stars. This, of course, made the principal star brighter than would ordinarily be used for parallax determinations. The

[1] Slocum, *Astrophysical Journal*, **41**, 237, 1915.

two stars were well separated when definition and guiding were both good.

The measures and calculations were carried out to 0.1 micron. The final values we turned into angle by multiplying by the value 1 mm = 20.″8.

The details of the plates follow.

TABLE 1

PLATES OF 70 OPHIUCHI

No.	Date	Hour Angle	Observers	Quality of Images
1	1914 May 2	$+0^h.4$	M	Good
6	May 9	−0.4	M	Fair-good
7	May 9	+0.2	M	Good
10	May 10	+0.3	M	Fair-good
13	May 11	−1.5	M	Fair-good
216	Sept. 9	+1.1	M	Fair
225	Sept. 13	+0.6	M, G	Poor
249	Sept. 15	+0.6	M	Fair-good
336	Sept. 28	+1.0	M	Fair
353	Sept. 30	+1.3	M	Fair-good
365	Oct. 1	+1.3	M, Ol	Fair
1119	1915 Mar. 27	−1.2	G	One good
1120	Mar. 27	−0.5	G	Good
1178	Apr. 11	0.0	A	Fair
1183	Apr. 12	−1.0	M	Fair
1203	Apr. 14	−0.9	A	Good
1213	Apr. 15	−0.9	M	Fair-good

M = S. A. Mitchell; Ol = Charles P. Olivier; A = Harold Alden; G = P. H. Graham.

COMPARISON STARS

No.	Diameter	X (Right Ascension)	Y (Declination)	Dependence
	mm	mm	mm	
1	0.12	−41.0	−23.2	+0.212
2	.12	−35.2	+35.0	.240
3	.10	+36.8	−29.5	.263
4	.13	+39.4	+17.7	+0.285
Principal star	.22	+ 3.7	+ 0.9	....
Companion	0.12	+ 3.8	+ 0.7	....

TABLE 2

REDUCTIONS FOR 70 OPHIUCHI, PRINCIPAL STAR

Plate	Solution (m)	Weight (p)	Parallax Factor (P)	Time in Days (t)	Residual (v)	$\sqrt{p}\cdot v$ in Arc
	mm				mm	
1	−0.0490	1.0	+0.752	−145	+0.0006	+0″.01
6	− .0486	0.9	+ .670	−138	− .0003	− .01
7	− .0492	1.0	+ .670	−138	+ .0003	+ .01
10	− .0500	0.9	+ .657	−137	+ .0010	+ .02
13	− .0474	0.9	+ .644	−136	− .0017	− .03
216	− .0578	0.6	− .980	− 15	− .0017	− .03
225	− .0582	0.4	− .990	− 11	− .0014	− .02
249	− .0582	0.9	− .995	− 9	− .0014	− .03
336	− .0600	0.7	− .998	+ 4	+ .0005	+ .01
353	− .0600	0.9	− .994	+ 6	+ .0005	+ .01
365	− .0624	0.7	− .992	+ 7	+ .0030	+ .05
1119	− .0449	0.7	+ .992	+184	+ .0007	+ .01
1120	− .0442	1.0	+ .992	+184	.0000	.00
1178	− .0471	0.7	+ .942	+199	+ .0026	+ .04
1183	− .0468	0.7	+ .929	+200	+ .0022	+ .04
1203	− .0432	1.0	+ .917	+202	− .0014	− .03
1213	−0.0418	0.9	+0.908	+203	−0.0029	−0.06

The normal equations are:

$$\begin{aligned} 13.9c + 3.155\,\mu + 3.7591\pi &= -0.7018 \\ +28.2249\,\mu + 4.8637\pi &= -0.1104 \\ +10.7958\pi &= -0.1184 \end{aligned}$$

from which

$$\begin{aligned} c &= -0.05255 \\ \mu &= +0.00075 = +0''.0157 \\ \pi &= +0.00699 = \mathbf{+0''.145 \pm 0''.007} \end{aligned}$$

Probable error corresponding to unit weight $\pm 0.00105 = \pm 0''.022$.

TABLE 3

REDUCTION FOR 70 OPHIUCHI, COMPANION

Plate	Solution (m)	Weight (p)	Parallax Factor (P)	Time in Days (t)	Residual (v)	$\sqrt{p}\cdot v$ in Arc
	mm				mm	
1	+0.0768	1.0	+0.752	−145	+0.0012	+0."02
6	+ .0808	0.9	+ .670	−138	− .0032	− .06
7	+ .0777	1.0	+ .670	−138	− .0001	.00
10	+ .0762	0.9	+ .657	−137	+ .0014	− .03
13	+ .0772	0.9	+ .644	−136	+ .0003	+ .01
216	+ .0678	0.6	− .980	− 15	+ .0014	+ .02
225	+ .0726	0.4	− .990	− 11	− .0034	− .04
249	+ .0682	0.9	− .995	− 9	+ .0011	+ .02
336	+ .0726	0.7	− .998	+ 4	− .0028	− .05
353	+ .0694	0.9	− .994	+ 6	+ .0005	+ .01
365	+ .0681	0.7	− .992	+ 7	+ .0018	+ .03
1119	+ .0896	0.7	+ .992	+184	+ .0027	+ .05
1120	+ .0928	1.0	+ .992	+184	− .0005	− .01
1178	+ .0935	0.7	+ .942	+199	− .0010	− .02
1183	+ .0919	0.7	+ .929	+200	+ .0005	+ .01
1203	+ .0934	1.0	+ .917	+202	− .0010	− .02
1213	+0.0925	0.9	+0.908	+203	−0.0001	0.00

The normal equations are:

$$\begin{aligned} 13.9\,c + 3.155\,\mu + 3.7591\pi &= +1.1193 \\ +28.2249\,\mu + 4.8637\pi &= +0.3895 \\ +10.7958\pi &= +0.3955 \end{aligned}$$

from which

$$\begin{aligned} c &= +0.07752 \\ \mu &= +0.''00377 = +0.''0782 \\ \pi &= +0.00794 = \mathbf{+0.''165 \pm 0.''007} \end{aligned}$$

Probable error corresponding to unit weight $\pm 0.00105 = \pm 0.''022$.

The equations for 70 Ophiuchi might have been solved by allowing for the orbital motion. The above-described photographs, however, were all taken within a year, and in this interval of time it was assumed that the orbital motion was linear, or in other words, proportional to the time. Orbital motion is, therefore, included in the determination of μ, the proper motion in 100 days.

Other determinations of the parallax of this system are:

Authority	Method	Principal Star A	Center of Gravity $\frac{AB}{2}$
Krueger	Heliometer	....	+0".156±0".010
Schur	Heliometer	....	+ .286± .031
Jewdokimov	Meridian circle	....	+ .279± .105
Flint	Meridian circle	0".19± 0".029	....
Slocum	Photography	0".212±0".007	....

Cygni 6 (19^h9^m, $+49°37'$)

This is a system of the 61 Cygni class with nearly common proper motion of 0".65 per year in position angle 344°. The stars are of type K, and of magnitudes 6.6 and 6.8. The rotating sector was used. According to Adams and Kohlschütter,[1] these stars, though separated by nearly 10", undoubtedly form a physical system, since they have radial velocities of −41 and −39 km respectively. The values of the parallaxes below and their common proper motion confirm this notion.

TABLE 1

PLATES OF CYGNI 6

Plate	Date		Hour Angle	Observers	Quality of Image
52	1914	May 30	-0^h8	M	Fair-good
53		May 30	−0.3	M	Good
58		May 31	−1.1	M	Fair
59		May 31	−0.7	M	Good
212		Sept. 7	−0.2	M	Good
237		Sept. 14	−0.1	M, A	Good
278		Sept. 21	−0.5	M	Good
279		Sept. 21	0.0	M	Good
291		Sept. 22	−0.4	M	Fair-good
292		Sept. 22	0.0	M	Good
346		Sept. 29	−0.1	M	Good
1184	1915	Apr. 12	−1.3	M	Fair
1185		Apr. 12	−1.0	M	Good
1196		Apr. 13	−1.1	Ol	Good
1204		Apr. 14	−1.2	A	Good
1205		Apr. 14	−0.9	A	Good

[1] *Astrophysical Journal*, **39**, 346, 1914.

COMPARISON STARS

No.	Diameter	X (Right Ascension)	Y (Declination)	Dependence
	mm	mm	mm	
1	0.15	−55.3	−27.8	+0.192
2	.20	−25.1	+41.2	+ .185
3	.10	+ 9.6	−32.7	+ .209
4	.10	+10.0	+32.6	+ .196
5	.21	+60.8	−13.3	+0.218
Parallax star, π_1	.18	+ 1.9	− 1.2	....
Parallax star, π_2	0.16	+ 2.1	− 0.9	....

TABLE 2

REDUCTION FOR CYGNI 6 (SEQ). BRIGHTER STAR

Plate	Solution (m)	Weight (p)	Parallax Factor (P)	Time in Days (t)	Residual (v)	$\sqrt{p}\cdot v$ in Arc
	mm				mm	
52	−0.0032	0.9	+0.608	−137	+0.0007	+0.″01
53	− .0020	1.0	+ .608	−137	− .0005	− .01
58	− .0008	0.7	+ .595	−136	− .0017	− .03
59	− .0035	1.0	+ .595	−136	+ .0010	+ .02
212	− .0094	1.0	− .852	− 37	+ .0004	+ .01
237	− .0082	1.0	− .906	− 30	− .0012	− .02
278	− .0094	1.0	− .948	− 23	− .0003	− .01
279	− .0103	1.0	− .948	− 23	+ .0006	+ .01
291	− .0103	0.9	− .952	− 22	+ .0006	+ .01
292	− .0116	1.0	− .952	− 22	+ .0019	+ .04
346	− .0081	1.0	− .978	− 15	− .0019	− .04
1184	− .0080	0.7	+ .990	+180	− .0030	− .05
1185	− .0112	1.0	+ .990	+180	+ .0002	.00
1196	− .0116	1.0	+ .988	+181	+ .0006	+ .01
1204	− .0115	1.0	+ .986	+182	+ .0004	+ .01
1205	−0.0120	1.0	+0.986	+182	+0.0009	+0.02

The normal equations are:

$$
\begin{aligned}
15.2\,c + 1.897\,\mu + 0.3689\pi &= -0.1271 \\
+22.5665\,\mu + 7.0162\pi &= -0.0655 \\
+11.9135\pi &= +0.0053
\end{aligned}
$$

from which

$$
\begin{aligned}
c &= -0.00805 \\
\mu &= -0.00299 = -0.''0622 \\
\pi &= +0.00245 = \mathbf{+0.''051 \pm 0.''006}
\end{aligned}
$$

Probable error corresponding to unit weight $\pm 0.00086 = \pm 0.''018$.

TABLE 3

REDUCTION FOR CYGNI 6 (PR). FAINTER STAR

Plate	Solution (m)	Weight (p)	Parallax Factor (P)	Time in Days (t)	Residual (v)	$\sqrt{p}\cdot v$ in Arc
	mm				mm	
52	+0.0541	0.9	+0.608	−137	+0.0018	+0.″04
53	+ .0543	1.0	+ .608	−137	+ .0016	+ .03
58	+ .0588	0.7	+ .595	−136	− .0021	− .05
59	+ .0570	1.0	+ .595	−136	− .0012	− .02
212	+ .0508	1.0	− .852	− 37	− .0012	− .02
237	+ .0475	1.0	− .906	− 30	+ .0018	+ .04
278	+ .0499	1.0	− .948	− 23	− .0009	− .02
279	+ .0481	1.0	− .948	− 23	+ .0009	+ .02
291	+ .0482	0.9	− .952	− 22	+ .0007	+ .01
292	+ .0474	1.0	− .952	− 22	+ .0015	+ .03
346	+ .0513	1.0	+ .978	− 15	− .0027	− .06
1184	+ .0468	0.7	+ .990	+180	− .0020	− .04
1185	+ .0452	1.0	+ .990	+180	− .0004	− .01
1196	+ .0446	1.0	+ .988	+181	+ .0002	.00
1204	+ .0432	1.0	+ .986	+182	+ .0015	+ .03
1205	+0.0446	1.0	+0.986	+182	+0.0001	0.00

The normal equations are:

$$
\begin{aligned}
15.2c + 1.897\,\mu + 0.3689\pi &= +0.7499 \\
+22.5665\,\mu + 7.0162\pi &= +0.0230 \\
+11.9135\pi &= +0.0130
\end{aligned}
$$

from which

$$
\begin{aligned}
c &= +0.04975 \\
\mu &= -0.00370 = -0.''0770 \\
\pi &= +0.00173 = \mathbf{+0.''036 \pm 0.''007}
\end{aligned}
$$

Probable error corresponding to unit weight $\pm 0.00107 = \pm 0.''022$.

Various determinations of the parallax of this system have been as follows:

Authority	Method	Brighter Star A	Fainter Star B	Center of Gravity $\frac{AB}{2}$
Ball	Equatorial (distance)	$+0''.504 \pm 0''.060$	...	...
Ball	Equatorial (pos. ang.)	$+0.383 \pm 0.13$	...	...
Ball	Equatorial (dist. and pos. ang.)	$+0.482 \pm 0.054$	...	...
A. Hall	Equatorial ($\Delta\delta$)	-0.094 ± 0.025	...	...
A. Hall	Equatorial ($\Delta\delta$)	-0.137 ± 0.017	...	...
A. Hall	Equatorial ($\Delta\alpha$)	$+0.023 \pm 0.009$	...	...
Chase	Heliometer	...	-0.027 ± 0.039	...
Kostinsky	Photography	$+0.040 \pm 0.03$	$+0.05 \pm 0.03$	$+0.04 \pm 0.02$
Russell	Photography	-0.011 ± 0.049	$+0.075 \pm 0.063$	...
Flint	Meridian circle	...	...	0.045 ± 0.021
Jewdokimov	Meridian circle	...	...	0.094 ± 0.055

Parallax work at the Leander McCormick Observatory was made possible, in as large quantities as was attempted during the past year, by the award to the writer by Columbia University of the Ernest Kempton Adams research fellowship. Grateful acknowledgment is hereby expressed. Appreciation is also due to the members of the observatory staff, Dr. Charles P. Olivier, Mr. Harold L. Alden, and Mr. P. H. Graham, for their hearty co-operation, and to Mr. R. C. Lamb, student in the University, for aid in computation.

Leander McCormick Observatory
University of Virginia
May, 1915

NOTE ON THE DENSITIES OF SECOND-TYPE STARS[1]

By HARLOW SHAPLEY

During the last few years our knowledge of stellar densities has been considerably increased through the acquisition of specific values for the mean density of certain classes of double stars; and, at the same time, there has been a growing need in studies of stellar development for more definite information regarding this as well as other physical properties of stellar bodies. From the study of eclipsing variables it is found that the average density of the first-type stars is one-tenth or two-tenths that of the sun. Moreover, the range of values is apparently limited, for from a total of some fifty carefully investigated binaries, no star of spectral type B or A is known to have less than one-hundredth the solar density. Among the second-type systems a number are known to be denser than the white stars; but, on the other hand, there are also solar-type stars of remarkably low density—so rare, in fact, that with solar mass their volumes must be hundreds of times that of the sun. This result has been stated in various recent articles, but only in a more or less summary fashion; and, as the existence of such low densities and the bearing they may have on current astronomical problems is not generally admitted, it is the object of the present communication to give the data upon which the conclusion depends in a manner sufficiently detailed to permit an easy inspection and consideration of its validity.

The point is of some importance in determining the order of stellar evolution. Admitting that stars in growing older contract and become denser, the stars of least density must obviously be the youngest; and, if we suppose that the order of evolution is uniformly that represented by the spectral sequence B, A, F, G, K, and M, we must expect the densities of second-type stars always to be greater than those of the first. In other words, second-type stars of low density find no place in this scheme of stellar evolution except by the assumption that these particular bodies, in spite of

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 107.

gravitation, expand as time goes on. There may be some other explanation of such small values—we might hypothecate the existence of important counter-gravitative forces, or deny the whole of the eclipse theory, or, regardless of the evidence, propose that these are exceptional bodies which do not develop in the normal manner or are preceded by as yet undiscovered whiter stars of still greater volume, rarity, and luminosity.[1] But otherwise, if the existence of the abnormal densities is proved, it would seem that the conventional order of the evolutionary stages must be modified.

The existence of many second-type stars of great intrinsic brightness is not questioned—for instance, Capella, the Cepheid variables, and the bright yellow stars in the Hyades group. With any reasonable assumption as to the surface brightness of these stars, the inevitable conclusion is either that they have enormous masses, or, if the masses are within the limits found in double-star systems, that the mean densities are extremely low. In some cases the dimensions must be so large that there can be no middle-ground adjustment that will keep both mass and density within the limits generally admitted in stellar studies. To account for the great size of the second-type stars of high luminosity, the choice between large mass or small density is generally made in favor of the former in order to maintain a late epoch in stellar development for these objects. But this implies for these stars, when earlier in their history they were of spectral type A or B, still greater dimensions than they now possess; and this circumstance, coupled with the much greater intrinsic light-emitting power of the whiter stars, would demand the presence of giant white forerunners of a magnitude, both absolute and apparent, not at present to be found.

This possible, though apparently improbable, interpretation of the great volume of isolated giant red and yellow stars contributes nothing to the question of whether the first-type stars are denser,

[1] In this connection it is well to keep in mind the group of B-type spectroscopic binaries whose periods exceed 100 days. We have as yet no assurance that their densities may not be peculiarly low. None of them is known to be an eclipsing variable. Of the second-type spectroscopic binaries, however, one-third have periods longer than the longest of the B's, so that, if length of period is to be the criterion for density, here again the white stars have intermediate values (*Mt. Wilson Contr.*, No. 99; *Astrophysical Journal*, **41**, 291, 1915).

in the mean and individually, than some of those of the second type; but, for valuable evidence, recourse may be had to the data of eclipsing binaries in which the mean density of each system can be determined independently of the mass.[1]

The questions to be answered are, first: Do these extremely low densities certainly exist? and secondly: Are the stars considered certainly those with the redder spectra? Since the evidence so far presented is by no means to be considered overwhelming, nor perhaps even incontrovertible, a specific record of the data is desirable in order to indicate the present status of the argument.

The equations giving the density in terms of the orbital elements have been developed in various forms, and results for 20 stars, which seem certainly to be of the second type, have been published. Some of these densities are abnormally low, but the fact that their derivation has been involved with the rather complicated orbital theory of binary systems perhaps detracts something from the conviction that the results would otherwise carry. There is, however, a simple relation which gives the upper limit of the mean density of an eclipsing binary without hypothesis as to the depth of the minima or the relative size or brightness of the components—in fact, in a circular orbit or one of small eccentricity the limit involves only the period and the duration of one eclipse. As the results from this relation are of the same order as those previously found, the conclusion as to the existence of very low densities is freed from the orbital theory and given a very direct derivation.

Expressing the period, P, and the semi-duration of eclipse, t, in days, and the mean density, d_0, in terms of that of the sun, we have (as shown in the supplementary note)

$$d_0 \leqq \frac{0.054}{P^2 \sin^3 \frac{2\pi t}{P}}. \tag{1}$$

Table I contains data relative to five eclipsing systems of low mean density. The upper limits in the fifth column were derived

[1] The assumption that isolated stars and binaries are physically comparable is necessary, of course, in applying to the general aggregate of stars the conclusions reached in the present case; but there seems to be no reason at present to doubt the validity of this assumption.

by the foregoing relation. The remaining data are from *Contributions from the Princeton University Observatory*, No. 3, pp. 82 ff., 1915.

TABLE I

DENSITIES AND DIMENSIONS OF FIVE ECLIPSING SYSTEMS

Star	Spectrum	Period	Semi-Duration of Minimum	Upper Limit Mean Density	Computed Density		Hypothetical Longest Radius		
					Bright Star	Faint Star	Bright Star	Faint Star	Relative Orbit
SX Cass...	G3	$36^d.572$	$2^d.5$	0.0005	0.0004	0.0002	15.3	18.6	59.
RX Cass...	K0	32.316	2.5	0.0005	0.0005	0.0004	14.8	14.8	53.
RZ Oph...	F8	261.9	8.0	0.00012	0.001	0.00003	10.1	33.5	217.
RT Lac....	G5	5.074	0.4	0.02	0.013	0.010	4.6	4.6	15.6
W Crucis..	Gp	198.5	22.0	0.00005	0.000002	0.000025	94.	36.	180.

Only for W Crucis are the observations adequate at present to give a curve and orbit of the first grade; the periods, however, are accurately known in all cases. To indicate how closely the duration of the eclipse can be ascertained, as well as to show how closely the variation of these stars resembles that of typical eclipsing binaries of shorter period, the computed light-curves are given in Figs. 1, 2, 3, and 4. That for W Crucis has already been published in this *Journal*;[1] the peculiarities of its spectrum and the small distance separating the components may entitle it to diminished weight in the present discussion.

The sources of the observations for the other four systems are as follows:

SX Cassiopeiae.—The measures are visual estimates by Luizet; the open circles represent points of low weight. Normal magnitudes are printed in *Contributions from the Princeton University Observatory*, No. 3, p. 132 (1915), and the orbit, *ibid.*, p. 86. The curve is computed from the uniform orbital elements.

RX Cassiopeiae.—Observations are by Wendell using a Harvard polarizing photometer, each point representing the measures of a single night. The curve is computed from the uniform elements (*ibid.*, pp. 86, 135).

[1] **36**, 148, 1912.

RZ Ophiuchi.—Only one-half of the curve is shown. The *x*'s are normal points from observations made at the Laws Observatory by Seares and Haynes; the dots define co-ordinates of the observed

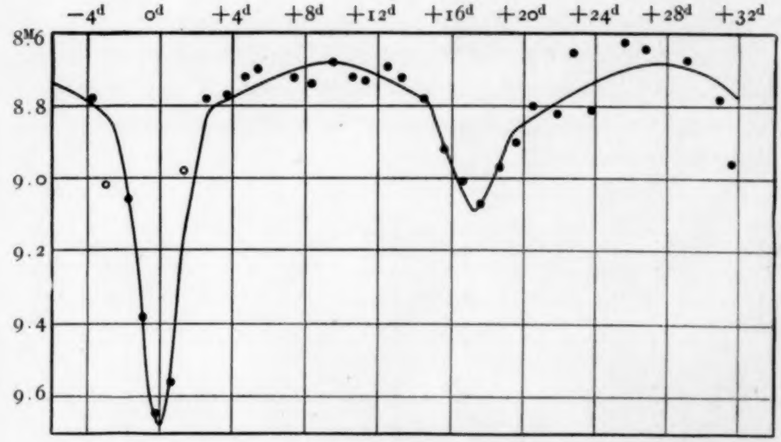


FIG. 1.—The light-curve of SX Cassiopeiae

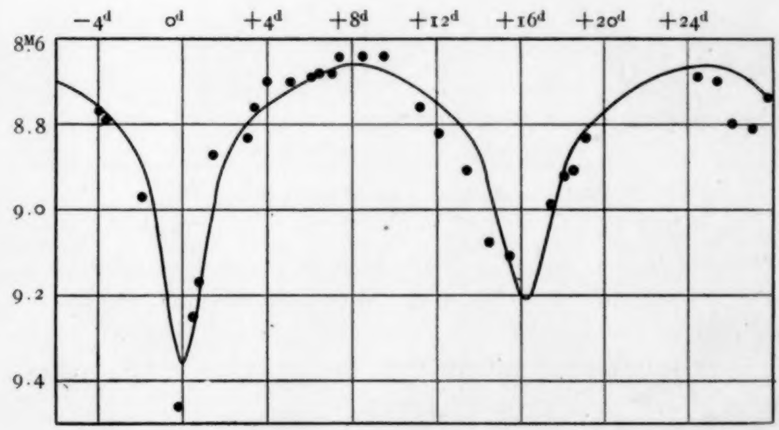


FIG. 2.—The light-curve of RX Cassiopeiae

curve by Nijland. The curve is computed from the darkened elements (*ibid.*, pp. 86, 157); the orbit is discussed in *Astronomische Nachrichten*, **194**, 225, 1913; see also *Laws Observatory Bulletin*, No. 16, 1908.

RT Lacertae.—The *x*'s represent normal magnitudes by Enebo; the dots, normals by Luizet (black squares have quadruple weight). The curve is computed from the darkened elements, the uniform solution being impossible (*Contributions from the Princeton University Observatory*, No. 3, pp. 17, 26, *et passim*).

One other variable similar to these—ϵ Aurigae—might be added to the list, but its marked peculiarities of spectrum make the interpretation of its light-curve uncertain.[1] Similarly β Lyrae must be considered anomalous.[2]

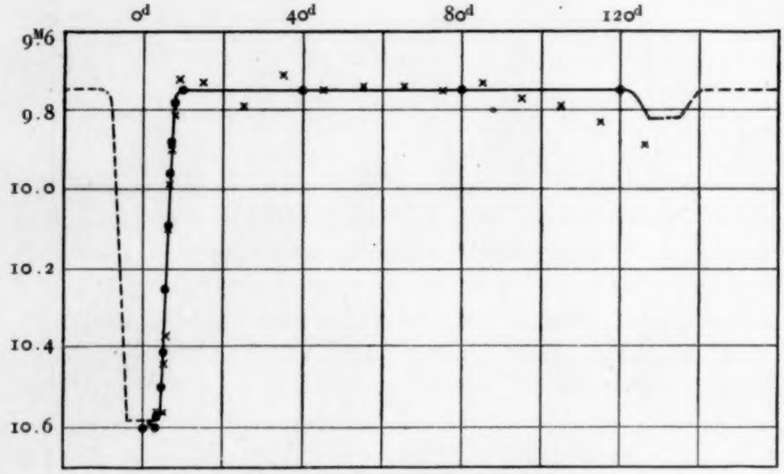


FIG. 3.—The light-curve of RZ Ophiuchi

The faintness of the stars in the table has made the classification of the spectra difficult.[3] To make certain, however, that they are definitely of the second type, three independent and accordant classifications of their spectra were kindly made at the writer's request at Harvard by Miss Cannon.

The tabulated semi-duration of the eclipse, which is used to compute the upper limit of the mean density, was read directly

[1] *Contributions from the Princeton University Observatory*, No. 3, pp. 20, 84, 94, 1915.

[2] *Ibid.*, pp. 71 ff.

[3] For the spectrum of W Crucis see *Astrophysical Journal*, 36, 153, 1912.

from the plot of the observations. For all but RZ Ophiuchi[1] the position and shape of the observed secondary minimum is sufficient to show that the orbital eccentricity is so small that the formula for the limiting density is valid. The depths and shapes of the secondary minima also show that the components must be of approximately the same dimensions, so that the mean density of the system also indicates the mean density of each component. This is not the case, however, for RZ Ophiuchi, where the steepness

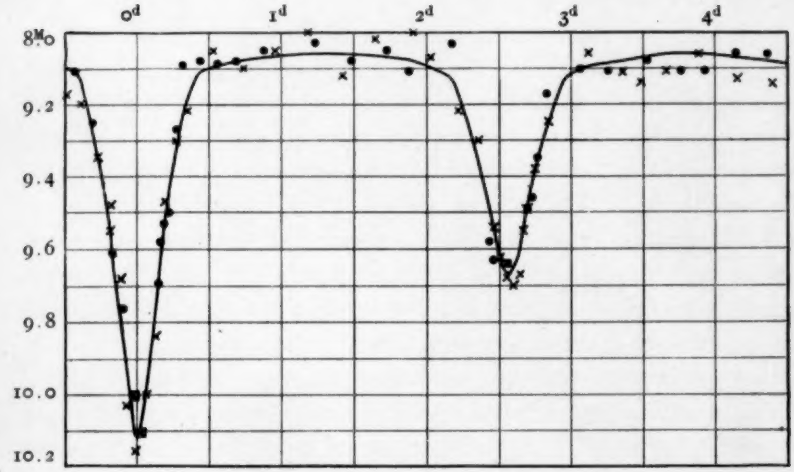


FIG. 4.—The light-curve of RT Lacertae

of the curve at primary eclipse shows that the component whose spectrum is classified as F8 is the smaller and brighter one for which the actual density, as computed from the orbital elements, is ten times the mean density; the second component is doubtless of a redder spectral type.

The computed densities in the fifth and sixth columns were derived from a discussion of the orbital elements and have been

[1] The deviation of the observation near the secondary minimum suggests a deeper secondary eclipse than computed, but one of the comparison stars used for this part of the curve at the Laws Observatory is suspected of slight variation. Graff's measures of the maximum light show no trace of a conspicuous secondary minimum (*Astronomische Nachrichten*, **176**, 79, 1907). A deeper secondary minimum would give a lower mean density for the brighter component. Data are now available for a revision of the orbit, which will be undertaken soon.

adjusted for polar flattening and probable irregular division of masses between the components in each system. They are close to the true values of the mean densities of these individual stars, but of course do not pretend to represent accurately the average density of the giant second-type stars as a whole. For a first-type star the lowest recorded density is 0.012, computed from a provisional curve for RZ Scuti, type B3; it is likely that further observation will tend to increase rather than diminish this value. The next lowest value is 0.017 for UZ Cygni, type A.[1]

Fig. 5 gives a diagrammatic representation of four of the foregoing low-density systems, the major axes of the several stars and orbits being taken from the last three columns of Table I. For convenience the linear dimensions of SX Cassiopeiae and RX Cassiopeiae have been divided by two, those of RZ Ophiuchi by four, and those of W Crucis by eight. (The tabulated dimensions for RT Lacertae show that it is intermediate between the giant systems and the sun.) The diagram shows not only that these binaries are typical in their relative dimensions, but also that the components are distinctly separated. If the components were in contact, as is probably the case with β Lyrae and RR Centauri,[2] we might expect the existence of a gaseous envelope that would give rise to spectral peculiarities; or, of more importance to the present problem, we might question the meaning of the computed densities, as Jeans[3] has done, because of possible interactions through a connecting neck of gas. But in these giant systems we find the stars as definitely and distantly separated as the average eclipsing and spectroscopic binary. The sun is drawn to scale (with the linear dimensions of the binaries reduced as mentioned above) on the assumption that each component of all the systems has solar mass. The masses are more likely to exceed the sun's than to be less. If each component is 8 times as massive as assumed,

[1] Stebbins has just announced that the mean density of δ Orionis (type B) is 0.006 (*Science*, N.S., **41**, 811, June 4, 1915; see also *Astrophysical Journal*, **42**, 144, 1915).

[2] *Monthly Notices*, **63**, 537, 548, 1903. Roberts finds the components to be actually overlapping in their lines of centers. The computed density, however, is entirely normal for a dwarf F-type system.

[3] *Astrophysical Journal*, **22**, 93, 1905.

the sun should be drawn one-half as large; if but one-eighth the solar mass, its diameter should be doubled. The true dimensions of the stars relative to the sun are probably well within these limits. For the purpose of comparison, the two eclipsing systems of highest known density, which are also of the second spectral type, are

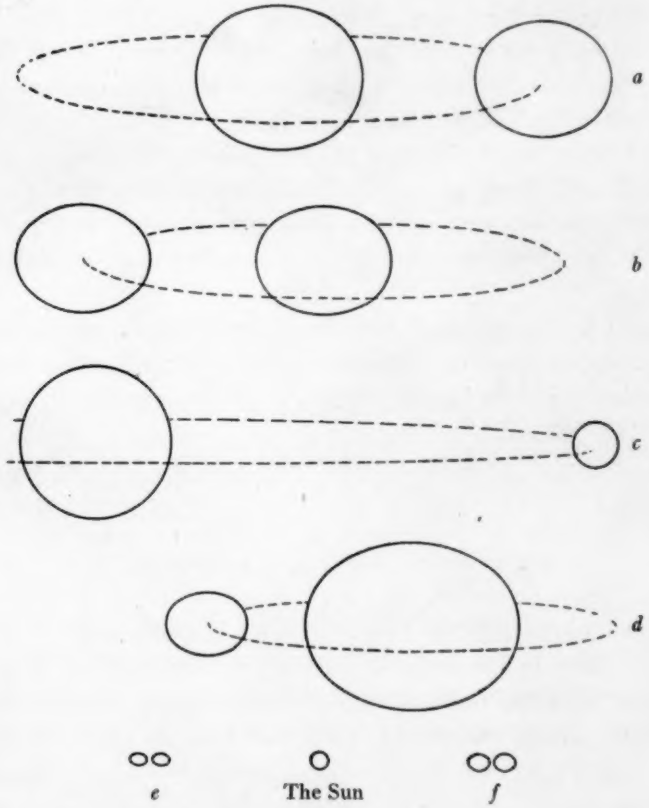


FIG. 5.—Eclipsing binaries of low and high density. The mass of each component is assumed equal to that of the sun.

a) SX Cassiopeiae; period $36^{d}.572$; spectrum G3. Reduced to one-half.
b) RX Cassiopeiae; period $32^{d}.316$; spectrum Ko. Reduced to one-half.
c) RZ Ophiuchi; period $261^{d}.9$; spectrum F8. Reduced to one-fourth.
d) W Crucis; period $198^{d}.5$; spectrum Gp. Reduced to one-eighth.
e) W Ursae Majoris; period $0^{d}.334$; spectrum G.
f) U Pegasi; period $0^{d}.375$; spectrum F?

included in the diagram, thus affording a fair illustration of giant and dwarf stars.

SUPPLEMENTARY NOTE ON THE DERIVATION OF EQUATION (1) AND SIMILAR RELATIONS

The equivalent of equation (1) has been given by Russell;[1] expressions for the lower limit of the mean density of a binary, involving only the period, have been given by Stebbins[2] and others. On the basis of what is now known of the elements of eclipsing star orbits the range can be much narrowed, and it may be of value to derive both of the limits from the beginning. The resulting simple relations will then give with considerable accuracy the mean density of an eclipsing variable from the two most easily observed quantities, namely, the length of the period and the duration of minimum.

Selecting the appropriate units of time, length, and density, we derive readily from the equation for elliptic motion the following expression for the mean density of a binary system:

$$d_0 = \frac{0.0134}{P^2(r_1^3+r_2^3)} = \frac{0.0134}{P^2(r_1+r_2)^3 X}, \tag{2}$$

in which

$$X = \frac{r_1^3+r_2^3}{(r_1+r_2)^3},$$

and P, r_1, r_2 are the revolution period and the radii of the component stars, relative to the distance between their centers. The ratio X has its maximum value, unity, when one of the radii is zero; its minimum, $\frac{1}{4}$, corresponds to $r_1 = r_2$. Hence,

$$\left.\begin{aligned} &\text{Upper limit of } d_0 = \frac{0.0537}{P^2(r_1+r_2)^3} \quad \text{(stars equal)} \\ &\text{Lower limit of } d_0 = \frac{0.0134}{P^2(r_1+r_2)^3} \quad \text{(one star a particle)} \end{aligned}\right\} \tag{3}$$

The stellar radii are determined only by a solution of the orbit, but in the above limiting expressions we may substitute quantities that are determined directly from the light-curve.

[1] *Astrophysical Journal*, **10**, 316, 1899. [2] *Ibid.*, **34**, 105, 1911.

For a circular orbit[1]

$$(r_1+r_2)^2=\cos^2 i \cos^2 \theta'+\sin^2 \theta', \tag{4}$$

where i measures the inclination of the orbit to the plane perpendicular to the line of sight and $\theta'=2\pi t/P$, t as before denoting the semi-duration of eclipse. For any given value of θ'

$$(r_1+r_2)^2=1 \quad \text{(maximum) for } i=0°$$
$$(r_1+r_2)^2=\sin^2 \theta' \quad \text{(minimum) for } i=90°$$

Hence,

$$\left.\begin{aligned}
&\text{Maximum upper limit } d_0=\frac{0.0537}{P^2 \sin^3 \theta'} && (r_1=r_2\,,\ i=90°)\\
&\text{Minimum upper limit } d_0=\frac{0.0537}{P^2} && (r_1=r_2\,,\ i=0°)\\
&\text{Maximum lower limit } d_0=\frac{0.0134}{P^2 \sin^3 \theta'} && (r_2=0\ ,\ i=90°)\\
&\text{Minimum lower limit } d_0=\frac{0.0134}{P^2} && (r_2=0\ ,\ i=0°)
\end{aligned}\right\} \tag{5}$$

The first of equations (5) is equivalent to (1).

If the stars are just in contact, $4t=P$, so that $\theta'=\pi/2$ and $(r_1+r_2)^3=1$ for all values of i. The limits then become independent of the inclination and

$$\text{Upper limit } d_0=\frac{0.0537}{P^2} \qquad (r_1=r_2)$$

$$\text{Lower limit } d_0=\frac{0.0134}{P^2} \qquad (r_2=0\ )$$

With the aid of known values of the orbital inclinations we can, in practice, raise the lower limits of d_0 as given by the last two of equations (5). Substituting (4) in (2) we obtain

$$d_0 \geqq \frac{0.0134}{P^2 (\cos^2 i \cos^2 \theta'+\sin^2 \theta')^{3/2}}\,, \tag{6}$$

which becomes an equality for one star a particle.

The average value of $\cos i$ for ninety eclipsing systems, all with ranges at primary eclipse in excess of 0.4 mag. (except β Aurigae,

[1] The necessary modifications of computations of density because of orbital eccentricity have been noted by Roberts (*Astrophysical Journal*, 10, 311, 1899) and others. In most cases the changes would be negligible for the present work.

range 0.1 mag. and cos $i=0.23$), is 0.112. On the average then (6) becomes[1]

$$d_0 > \frac{0.0134}{P^2(0.013+0.987\sin^2\theta')^{3/2}}\,. \tag{7}$$

For three systems only does cos i exceed 0.4, and in but one exceptional case is the inclination less than 60°. A very safe limit, therefore, is cos $i \leqq 0.5$, and then we have finally

$$d_0 > \frac{0.108}{P^2(1+3\sin^2\theta')^{3/2}}\,. \tag{8}$$

In equations (1) and (8) we have pretty close limits for the mean density which depend not at all on orbital elements. If on the basis of the depths and character of primary and secondary eclipses we can assume equal components, then $X=\frac{1}{4}$ and the right-hand member of (8) may be multiplied by four; and, at any rate, since only in the exceptional case of $r_2 < 0.25r_1$ is X greater than one-half, this limit of density may be safely doubled in 95 per cent of the eclipsing systems.

Mount Wilson Solar Observatory
May 1915

[1] Similarly the average value of (1) may be derived, and is found to be just four times as large as (7).

Reviews

Dialogues concerning Two New Sciences. By GALILEO. Translated from the Italian and Latin into English by HENRY CREW and ALFONSO DE SALVIO. With an introduction by ANTONIO FAVARO. New York: Macmillan, 1914. Pp. 300. $2.00.

"For more than a century English-speaking students have been placed in the anomalous position of having Galileo constantly referred to as the founder of modern physical science, without having any chance to read, in their own language, what Galileo himself has to say." With this pertinent remark Messrs. Crew and De Salvio preface their truly excellent translation of *The Two New Sciences.* In this, the last and greatest of his works, Galileo laid the foundations of two new subjects—strength of beams and uniformly accelerated motion. The book is in the form of dialogues between Salviati, a student of Galileo, the Academician, and Sagredo and Simplicimus, the one a broad-minded seeker after truth, the other an uncompromising Peripatetic. The dialogues are divided into four days, after the manner of chapters, but so numerous and delightful are the digressions, ranging over the broad field of physics, that the whole seems rather a conversation, naturally developed, than a carefully worked-out treatise.

Salviati undertakes to explain why similar machines, of the same material and with parts of proportional dimensions, cannot bear proportional strains. This leads to a consideration of cohesion, of vacua, of rarefactions and condensations, of resistance of media to falling bodies, of the laws of the pendulum; and the day closes with a digression on the laws of vibrating strings. The Aristotelian hypothesis that, in a given medium, heavy bodies fall with speeds proportional to their weights is disproven by reason as well as by experiment. Suppose two bodies, one much the heavier, be tied together. If the old idea were correct, then the lighter would retard the heavier. But this would mean that the two together, though heavier than the heavy one alone, would fall more slowly than the heavy one alone—a conclusion contrary to the hypothesis. Therefore the assumed hypothesis is false. By actual experiment it is found that bodies fall with very nearly equal speeds, and the theory is advanced that these inequalities are due to resistance of the air. Two methods are mentioned for determining the weight or specific gravity of

the air. One is especially simple. Water is forced into a vessel until the imprisoned air is under considerable pressure; the vessel is weighed; the imprisoned air is allowed to escape; the vessel is again weighed. Then the difference in weight is the weight of a volume of air equal to the volume of water in the vessel.

The second day's discourse is confined to strength of beams. The elements of this subject are established by simple geometrical reasoning from the law of the lever.

On the third and fourth days is read and discussed a book on *Motion*, written by the Academician Galileo. The first part treats briefly of uniform motion, and more at length of uniformly accelerated motion, which is for the first time precisely defined. The laws of falling bodies are demonstrated by means of inclined planes and a water clock. Then follows a geometrical treatment of motion on inclined planes, the essentials of which are found almost unchanged in the textbooks on physics of today.

The second part of this book deals with trajectories. Resistance to the air is disregarded as relatively slight and uncertain. The resultant of a uniform horizontal motion and the uniformly accelerated motion of a falling body is shown to be a semi-parabola. In the motion of projectiles, the horizontal component of velocity is measured by the height from which a body must fall to attain that particular velocity and is called the "sublimity." Tables are computed, giving the amplitudes, altitudes, and sublimities of trajectories for a given muzzle velocity and varying angles of elevation.

The dialogues afford an intimate acquaintance with conditions of the days of Galileo. Geometry was the mathematical instrument, and in the hands of a Galileo did surprising service. Nomenclature was obscure, ideas confused, and, above all, the experimental method was new. By a nice adjustment of literalness and freedom, the translators have retained the contemporary atmosphere without in any way sacrificing clearness.

The book is further enriched with an introduction by Antonio Favaro, of the University of Padua, editor of the Italian national edition of Galileo's works; with an excellent portrait of Galileo, as a frontispiece; and with a facsimile of the title-page to the Elzevir edition of 1638. Large print and good paper add materially to the book's attraction.

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